Discrete Grey Forecast Modelling: Principle, Models and Further Studies

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Contents

1 Principle of grey forecast modelling

2 Discrete grey forecast models

3 Further studies of grey forecasting models
Introduction

“Grey system”, Deng Julong, 1982

Black system

Grey system

White system

Number covered interval set can be known, the real number is unknown

Grey

Number sequence is very short, could not satisfy the demand of Statistic theory
Introduction

“Grey forecasting”, 1983
“Grey forecast model”, 1984

- GM (1, 1), single-variable grey dynamic model
- GM (1, N), multi-variable grey dynamic model

\[ \frac{dx_{1}}{dt} + ax_{1} = b \]

\[ x(0)(k) + az(1)(k) = b \]

Principle of grey forecast modelling
<table>
<thead>
<tr>
<th>Country or Region</th>
<th>Country or Region</th>
<th>Country or Region</th>
<th>Country or Region</th>
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<td>TAIWAN</td>
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<td>INDIA</td>
<td>NETHERLANDS</td>
<td>ROMANIA</td>
<td>SWEDEN</td>
<td>WALES</td>
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<td>DENMARK</td>
<td>IRAN</td>
<td>NEW ZEALAND</td>
<td>RUSSIA</td>
<td>SWITZERLAND</td>
<td></td>
</tr>
</tbody>
</table>

**Contents**

1. Principle of grey forecast modelling
2. Discrete grey forecast models
3. Further studies of grey forecasting models
Theoretical breakthroughs of GFM

Up to present, GM(1,1) model is the most famous forecasting model in grey system theory and it has been used in various fields.

Continuous equation

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = b$$

Discrete equation

$$x^{(0)}(k) + az^{(1)}(k) = b$$

- Literatures shows that GM(1,1) model can produce high precision in a large amount cases, but sometime the results is not very good.
- What’s the problem?
- In our opinion, the main point is that in the process of creating a GM (1, 1) model, estimating parameters adopted a discrete equation while calculating simulative and predictive values adopted a continuous equation. The difference between the two equations resulted in low simulative and forecasting precision.
- The discrete grey forecasting model (DGM) is constructed to solve such problem.
- In DGM, the accumulating generation of original sequence is keep. The parameter estimation and simulative value are calculated with the same equation.
Theoretical breakthroughs of GFMs

**Definition 1** Assume that the sequence

\[ X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)\} \]

is an original data sequence, the sequence

\[ X^{(1)} = \{x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)\} \]

is the accumulated generation sequence of \( X^{(0)} \), where

\[ x^{(2)}(k) = \sum_{i=1}^{k} x^{(1)}(i), k = 1, 2, \ldots, n \]

The equation

\[ x^{(1)}(k+1) = \beta_1 x^{(1)}(k) + \beta_2 \]

is called discrete grey model abbreviated as DGM (1,1).

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**Time response equation of GM(1,1) model**

\[ \hat{x}^{(1)}(k+1) = (x^{(0)}(1) - \frac{b}{a})e^{-\alpha k} + \frac{b}{a}, k = 1, 2, \ldots, n-1 \]

**Recursive function of DGM model**

\[
\begin{cases}
\hat{x}^{(1)}(k+1) = \beta_1 (\hat{x}^{(1)}(1) - \frac{\beta_2}{1-\beta_1}) + \frac{\beta_2}{1-\beta_1}, k = 1, 2, \ldots, n-1 \\
\hat{x}^{(1)}(1) = x^{(0)}(1) + \beta_2
\end{cases}
\]
Theoretical breakthroughs of GFMs

Therefore, when the value of $\alpha$ is less, $\beta_1 \approx e^{-\alpha}$, substitute $e^{-\alpha}$ with $\beta_1$, then the result of GM(1,1) model is infinite approximate to the result of DGM model. Therefore, DGM model and GM(1,1) model can be consider as the different expressions of the same model. When the value of $\alpha$ is less, DGM model and GM(1,1) model can substitute each other.

when the value of $\alpha$ is not less, GM(1,1) model and DGM model are different.


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Theoretical breakthroughs of GFMs

GM model of multi-variables

\[
\frac{d^n x_i^{(1)}}{dt^n} + a_1 \frac{d^{n-1} x_i^{(1)}}{dt^{n-1}} + a_2 \frac{d^{n-2} x_i^{(1)}}{dt^{n-2}} + \cdots + a_n x_i^{(1)} = b_{ij} x_j^{(1)} + b_{kn} x_k^{(1)} + b_k
\]

Multi-variable DGM model

\[
\alpha^{(n)} x_i^{(1)}(k) + \sum_{j=1}^{m} a_j^{(n)} x_j^{(1)}(k) + a_{n+1}^{(n)}(k) = \sum_{j=1}^{m} b_j x_j^{(1)}(k) + b_k
\]

Similar with GM(1,1) and single variable DGM model, GM(n, h) model and multi-variable DGM model are equal to each other through data transformation.

Case A: Mobile telecommunication costumer number forecasting-DGM(1,N)

Recently, Mobile telecommunication rapidly increased in China. The mobile company need to forecast customer amount to arrange devices and make service strategy.

Current models mainly focus on single variable forecasting while did not consider influent factors.

<table>
<thead>
<tr>
<th>Year</th>
<th>Customer amount (万户)</th>
<th>Mobile device amount (万户)</th>
<th>Total revenue (亿元)</th>
<th>Population (万人)</th>
<th>Long distance device (万端)</th>
<th>Local device (万门)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>8455.3</td>
<td>13905.6</td>
<td>126745</td>
<td>99214.6</td>
<td>565.4964</td>
<td>17825.6</td>
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<tr>
<td>2001</td>
<td>14522.2</td>
<td>21926.3</td>
<td>127647</td>
<td>109655.2</td>
<td>703.5789</td>
<td>25566.3</td>
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<td>2002</td>
<td>20600.5</td>
<td>27400.3</td>
<td>130451</td>
<td>120332.7</td>
<td>771.3102</td>
<td>28656.9</td>
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<td>2003</td>
<td>26995.3</td>
<td>33698.4</td>
<td>135822.8</td>
<td>135822.8</td>
<td>809.5986</td>
<td>35082.5</td>
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<tr>
<td>2004</td>
<td>33482.4</td>
<td>39684.3</td>
<td>159878.3</td>
<td>139227</td>
<td>1262.9984</td>
<td>42346.9</td>
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<tr>
<td>2005</td>
<td>39340.6</td>
<td>48241.7</td>
<td>184937.4</td>
<td>144084.7</td>
<td>1371.6211</td>
<td>47158.1</td>
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<tr>
<td>2006</td>
<td>46105.8</td>
<td>58582.8</td>
<td>216314.4</td>
<td>151860</td>
<td>1492.2431</td>
<td>50279.9</td>
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<td>2007</td>
<td>54730.6</td>
<td>68580.1</td>
<td>246599.3</td>
<td>160510.8</td>
<td>1599.2211</td>
<td>530514.3</td>
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<td>2008</td>
<td>64124.5</td>
<td>85496.1</td>
<td>265810.3</td>
<td>163802</td>
<td>1690.7199</td>
<td>568652.2</td>
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<td>2009</td>
<td>73737.4</td>
<td>99298.7</td>
<td>28656.9</td>
<td>1641464</td>
<td>1708.9961</td>
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<tr>
<td>2010</td>
<td>85080.3</td>
<td>150284.9</td>
<td>340902.8</td>
<td>1641464</td>
<td>1803.9961</td>
<td>64837.3</td>
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</tbody>
</table>
Case A: Mobile telecommunication costumer number forecasting-DGM(1,N)

We construct Grey relational models between X1 and other variables. As show in the table, X2 and X3 are strongly related with X1 while other 3 variables are not.

2000-2004, Mobile device amount mainly drive the increasing trend. While after 2005, the GDP became the main drive variable.

<table>
<thead>
<tr>
<th>年份</th>
<th>Mobile device amount(万户) $X_2$</th>
<th>GDP(亿元) $X_1$</th>
<th>Population(万人) $X_4$</th>
<th>Long distance device(万端) $X_5$</th>
<th>Local device(万门) $X_6$</th>
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<tbody>
<tr>
<td>2000-2004</td>
<td>0.80</td>
<td>0.61</td>
<td>0.58</td>
<td>0.65</td>
<td>0.69</td>
</tr>
<tr>
<td>2005-2009</td>
<td>0.64</td>
<td>0.94</td>
<td>0.64</td>
<td>0.72</td>
<td>0.67</td>
</tr>
</tbody>
</table>

DGM(1, N)

\[ x_1^{(1)}(k) - 0.63392x_1^{(1)}(k - 1) = -0.019643x_2^{(1)}(k) + 0.10377x_3^{(1)}(k) - 3350 \]
## Case A: Mobile telecommunication customer number forecasting - DGM(1,N)

<table>
<thead>
<tr>
<th>Year</th>
<th>Amount</th>
<th>DGM(1,1)</th>
<th>DGM(1,N)</th>
<th>Linear Regression</th>
<th>GM(1,N)</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td>Value</td>
<td>Error (%)</td>
<td>Value</td>
<td>Error (%)</td>
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<tr>
<td>2000</td>
<td>8453.3</td>
<td>8453.3</td>
<td>0</td>
<td>8453.3</td>
<td>0</td>
</tr>
<tr>
<td>2001</td>
<td>14522.2</td>
<td>18914</td>
<td>30.2</td>
<td>14524</td>
<td>0.1</td>
</tr>
<tr>
<td>2002</td>
<td>20600.5</td>
<td>22565</td>
<td>9.5</td>
<td>21155</td>
<td>2.7</td>
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<tr>
<td>2003</td>
<td>26995.3</td>
<td>26920</td>
<td>0.3</td>
<td>26842</td>
<td>0.6</td>
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<td>32115</td>
<td>4.1</td>
<td>32826</td>
<td>2</td>
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<td>2005</td>
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</tr>
<tr>
<td>2006</td>
<td>46105.8</td>
<td>45708</td>
<td>0.9</td>
<td>46003</td>
<td>0.3</td>
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<td>2007</td>
<td>54730.6</td>
<td>54530</td>
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<td>0.6</td>
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<td>1.8</td>
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<td>2010</td>
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<td>92589</td>
<td>7.8</td>
<td>85523</td>
<td>0.5</td>
</tr>
</tbody>
</table>

### GM model based on interval grey sequence

- **Fig. Simulative and predicative value of grey number sequence**
- **Definition of grey numbers**
- **Operations of grey numbers**
- **Simulative and predicative value based on optimized methods**
**Definition 2** The whole character information of an object is called proposition-information field or information background of proposition which is marked as $\mathcal{F}(\Theta)$.

\[
\begin{align*}
\text{Interval} &\quad \text{Grey number} \\
[a, b] &\quad [a, b]
\end{align*}
\]

<table>
<thead>
<tr>
<th>Assumed Real number</th>
<th>Grey numbers</th>
<th>Information background</th>
<th>Grey degree</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3</td>
<td>[2,4]</td>
<td>[0,10]</td>
</tr>
<tr>
<td>B</td>
<td>5</td>
<td>[2,8]</td>
<td>[0,20]</td>
</tr>
<tr>
<td>A+B=C</td>
<td>8</td>
<td>[4,12]</td>
<td>[0,30]</td>
</tr>
</tbody>
</table>

A, B: Simple grey number
C: Integrated grey number
GM model based on interval grey sequence

**Definition 3**  Due to some proposition \( \mathcal{P} \), \( \mathcal{P}(\theta) \) is the proposition-information field. For proposition is expressed incompletely or people is difficult to catch all the information of the proposition we can merely know the probable range of the proposition’s value or a set including several values. Marked the set as \( D \). So the proposition can be expressed as an uncertainty number \( o \) under the \( \mathcal{P}(\theta) \). \( o' \) is the true value of the proposition. Then we call

1. \( o \) is a grey number under the proposition \( \mathcal{P} \).
2. \( D \) is the value-covered set of \( o \).

---

GM model based on interval grey sequence

**Definition 4**  Let \( D_i \) and \( o' \) as the value-covered set and true value of grey number \( o \). Let \( D_j \) and \( o' \) as the value-covered set and true value of grey number \( o_j \). \( \circ \) as an operation. Let \( o_i \) as the result of \( o \) and \( o_j \) on the \( \circ \) operation. Let \( D_k \) as the value-covered of the grey number \( o_k \). Then we have the general \( \circ \) operation formula.

\[
\circ_i \circ_j = \circ_k \iff \{ \forall o' \in D_i, o' \in D_j, \exists o' \in D_k : o' \circ o' = o_k, D_i \circ D_j = D_k \}
\]

abbreviated as \( \circ_i \circ_j = \circ_k \).

Due to the grey numbers \( o_i \) and \( o_j \), their true values \( o_i' \) and \( o_j' \) are unknown. So the most important operation is the operation of \( D_i \) and \( D_j \) in the general \( \circ \) operation formula.
**GM model based on interval grey sequence**

**Definition 5** Suppose grey numbers \( \overline{a}_i, \overline{b}_i, \cdots, \overline{c}_i \) have the corresponding continuous covered sets \( D_i = [a_i, b_i] \). If \( \overline{0} = f(\overline{0}, \overline{0}, \cdots, \overline{0}, \overline{0}) \) is the result under two or more \( \circ \) operations, \( \circ \in \{+, -, \cdot, \div\} \). Then the corresponding value-covered set \( D = f(D_1, D_2, \cdots, D_n) \) of complex number \( \overline{0} \) can be calculated by two optimized model:

\[
\begin{align*}
\min f(x_1, x_2, \cdots, x_n, \overline{0}) & \quad \max f(x_1, x_2, \cdots, x_n, \overline{0}) \\
\text{s.t.} & \begin{cases}
\alpha_i \leq x_i \leq \beta_i \\
\vdots \\
\alpha_n \leq x_n \leq \beta_n
\end{cases} & \begin{cases}
\alpha_i \leq x_i \leq \beta_i \\
\vdots \\
\alpha_n \leq x_n \leq \beta_n
\end{cases}
\end{align*}
\]

Let \( a = \min f(x_1, x_2, \cdots, x_n, \overline{0}), b = \max f(x_1, x_2, \cdots, x_n, \overline{0}) \). Then the corresponding value-covered set \( D = f(D_1, D_2, \cdots, D_n) = [a, b] \).

---

**GM model based on interval grey sequence**

**Definition 6** Assume that the sequence

\[X^{(0)}(\overline{0}) = \{x^{(0)}(\overline{0}_1), x^{(0)}(\overline{0}_2), \cdots, x^{(0)}(\overline{0}_n)\}\]

is original grey number sequence, \( x^{(0)}(\overline{0}_i), i = 1, 2, \cdots, n \) is interval grey number. Define \( x^{(0)}(\overline{0}_i) \in [a_i, b_i], x^{(0)}(\overline{0}_1), \cdots, x^{(0)}(\overline{0}_n) \in [a_n, b_n] \)

\[X^{(0)}(\overline{0}) = \{x^{(0)}(\overline{0}_1), x^{(0)}(\overline{0}_2), \cdots, x^{(0)}(\overline{0}_n)\}\]

is the accumulated generation sequence of \( X^{(0)}(\overline{0}) \), where

\[x^{(0)}(\overline{0}_k) = \sum_{i=1}^{k} x^{(0)}(\overline{0}_i), k = 1, 2, \cdots, n\]

Then the equation

\[
\begin{align*}
[\bar{x}^{(0)}(\overline{0}_1)] &= \alpha_1 x^{(0)}(\overline{0}_1) + \beta_1 \\
[\bar{x}^{(0)}(\overline{0}_2)] &= \alpha_2 x^{(0)}(\overline{0}_2) + \beta_2
\end{align*}
\]

is called interval grey number sequence discrete grey model abbreviated as IG-DGM.
Theorem 1 According to the least squares estimate sequence

\[ \hat{\beta} = [\beta_1, \beta_2] = (B^T B)^{-1} B^T Y \]

Set \( \tilde{x}^{(i)}(\bar{\Theta}_k) = x^{(i)}(\bar{\Theta}_k) + \beta_1 \), then recursive function is given by

\[ \tilde{x}^{(i)}(\bar{\Theta}_{k+1}) = \beta_1 \tilde{x}^{(i)}(\bar{\Theta}_k) + \frac{1 - \beta_1}{1 - \beta_2} \beta_2, k = 1, 2, \ldots, n - 1 \]

or

\[ \tilde{x}^{(i)}(\bar{\Theta}_{k+1}) = \beta_1 \tilde{x}^{(i)}(\bar{\Theta}_k) - \frac{\beta_2}{1 - \beta_1} \tilde{x}^{(i)}(\bar{\Theta}_k) + \frac{\beta_2}{1 - \beta_1}, k = 1, 2, \ldots, n - 1 \]

Theorem 2 The restored values of \( \tilde{x}^{(i)}(\bar{\Theta}_k) \) can be given by

\[ \tilde{x}^{(i)}(\bar{\Theta}_{k+1}) = x^{(i)}(\bar{\Theta}_{k+1}) = \tilde{x}^{(i)}(\bar{\Theta}_k) - \tilde{x}^{(i)}(\bar{\Theta}_k), k = 1, 2, \ldots, n - 1. \]

Substitute \( \tilde{x}^{(i)}(\bar{\Theta}_k) \) and \( x^{(i)}(\bar{\Theta}_k) \), we can get

\[ \tilde{x}^{(i)}(\bar{\Theta}_k) = x^{(i)}(\bar{\Theta}_k) - x^{(i)}(\bar{\Theta}_k) = (\beta_1^{k+1} - \beta_1^k) \tilde{x}^{(i)}(\bar{\Theta}_k) - \frac{\beta_2}{1 - \beta_1} \]

Theorem 3, the maximum and minimum simulate value of \( \tilde{x}^{(i)}(\bar{\Theta}_k) \) can be get from the optimized model

\[ \begin{align*}
\text{max} x^{(i)}(\bar{\Theta}_k) &= x^{(i)}(\bar{\Theta}_k) + \beta_1 \\
\text{max} x^{(i)}(\bar{\Theta}_k) &= x^{(i)}(\bar{\Theta}_k) + \beta_1 \\
\text{max} x^{(i)}(\bar{\Theta}_k) &= x^{(i)}(\bar{\Theta}_k) + \beta_1 \\
\text{max} x^{(i)}(\bar{\Theta}_k) &= x^{(i)}(\bar{\Theta}_k) + \beta_1 \\
\end{align*} \]

and

\[ \begin{align*}
\text{min} x^{(i)}(\bar{\Theta}_k) &= x^{(i)}(\bar{\Theta}_k) + \beta_1 \\
\text{min} x^{(i)}(\bar{\Theta}_k) &= x^{(i)}(\bar{\Theta}_k) + \beta_1 \\
\text{min} x^{(i)}(\bar{\Theta}_k) &= x^{(i)}(\bar{\Theta}_k) + \beta_1 \\
\text{min} x^{(i)}(\bar{\Theta}_k) &= x^{(i)}(\bar{\Theta}_k) + \beta_1 \\
\end{align*} \]
**GM model based on interval grey sequence**

Theorem 4 The maximum and minimum simulate value of $x^0(t), t = 2, 3, \ldots, n$ and the forecasting value $x^0(t), t = n+1, n+2, \ldots$ can be got from the optimized model

$$
\begin{align*}
\max x^0(t) &= (A_i - A_i^n)x^0(t) + A_i \\
\min x^0(t) &= (A_i - A_i^n)x^0(t) + A_i
\end{align*}
$$

and

$$
\begin{align*}
\max x^0(t) &= (A_i - A_i^n)x^0(t) + A_i \\
\min x^0(t) &= (A_i - A_i^n)x^0(t) + A_i
\end{align*}
$$

**Case B: Interval grey forecasting—IG-DGM**

Numerical Example:

We construct Optimized IN-DGM model based on interval grey numbers sequence.

And construct DGM(1,1) model and GM(1,1) model separately with upper and lower values.

<table>
<thead>
<tr>
<th></th>
<th>Upper</th>
<th>Lower</th>
<th>[ ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>21.1</td>
<td>19.3</td>
<td>[19.3, 21.1]</td>
</tr>
<tr>
<td>2</td>
<td>26.5</td>
<td>24.5</td>
<td>[24.5, 26.5]</td>
</tr>
<tr>
<td>3</td>
<td>36.1</td>
<td>33.3</td>
<td>[33.3, 36.1]</td>
</tr>
<tr>
<td>4</td>
<td>52.3</td>
<td>48.7</td>
<td>[48.7, 52.3]</td>
</tr>
<tr>
<td>5</td>
<td>78.8</td>
<td>74.6</td>
<td>[74.6, 78.8]</td>
</tr>
</tbody>
</table>
Case B: Interval grey forecasting—IG-DGM

\[ S_i \in [\bar{y_i} \pm \bar{y_i}] \]

\[ \bar{S_i} \in [\bar{y_i} \pm \bar{y_i}] \]

\[ (PSVSC(\%)) = 1 - A \div B \]

Percent of simulating value set covered (PSVSC(\%))

<table>
<thead>
<tr>
<th>No.</th>
<th>Actual value</th>
<th>Simulate value of IN-NDGM model</th>
<th>PSVSC (%)</th>
<th>APEM (%)</th>
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</thead>
<tbody>
<tr>
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<td>dU</td>
<td>Mean</td>
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MPSVSC (%) = 1.73
MAPEM (%) = 0.14
Case B: Interval grey forecasting—IG-DGM

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<td>77.69</td>
</tr>
</tbody>
</table>

MPSVSC (%) = 35.88
MAPEM (%) = 2.70


Contents

1. Principle of grey forecast modelling
2. Discrete grey forecast models
3. Further studies of grey forecasting models
NO.1 Modelling sequence condition analysis

- What are the real sequences developing law for different grey forecasting models?
- Whether the errors between the real value and the simulate value of modelling sequences have a special distribution?
- How many of the data should be included in modelling process?
- How to choose the suitable sequence interval to construct the grey forecasting models?

NO.2 Grey forecasting models based on grey information

- Grey number operation
- Modelling
- Solution
- Simulation
- Test
- Real application
NO.3 Grey forecasting models for special usage

- limited upper value forecasting
- shock wave system forecasting
- system structure forecasting

Forecasting problem with the limited upper value

Throughput of airdrome, port
Traffic flow of highway, bridge or city transportation system
Power system generating capacity

Sequence X
Generated sequence XD

Stage 1 Stage 2 Stage 3

System state A
System state B

Arrest from outside
Proportion assign

NO.4 Compare Grey forecast models

- Compare GMs
- Compare GMs with other forecasting models (ARIMA, LR, …)
Thank You!