Quantification of Perception Clusters Using R-Fuzzy Sets and Grey Analysis

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Outline for the Presentation

1. Introduction
2. R-Fuzzy Sets
3. The Significance Measure
4. Grey Relational Analysis
5. Observations
6. Conclusion

This presentation will...

- Describe the concept of an *R-fuzzy* set, first proposed by Yang and Hinde
- Present the *significance measure* for the quantification of R-fuzzy sets
- Describe the notion of *grey analysis* to cater for an additional level of inspection, based on the *absolute degree of grey incidence*
- Propose a *new framework* for perception analysis and quantification
- Demonstrate the *enhanced* framework through a worked example

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Approximations Preliminaries

An Information System

Assume that $\Lambda = \left(\mathbb{U}, A\right)$ is an information system, and that $B \subseteq A$ and $X \subseteq \mathbb{U}$. One can approximate set X with the information contained in B via a lower and upper approximation set.

- The lower approximation is the set of all objects that *absolutely* belong to set X with respect to B
- It is the union of all equivalence classes in $[x]_B$ which are *contained* within the target set X
- The upper approximation is the set of all objects which can be classified as being *possible* members of set X with respect to B
- It is the union of all equivalence classes that have a *non-empty intersection* with the target set X

Approximations Preliminaries

- The *lower approximation* is given by the formal expression:

$$\underline{B}X = \left\{ x \;\middle|\; [x]_B \subseteq X \right\}$$

$$\underline{B}(x) = \bigcup_{x \in \mathbb{U}} \left\{ B(x) : B(x) \subseteq X \right\}$$

- The *upper approximation* is given by the formal expression:

$$\overline{B}X = \left\{ x \;\middle|\; [x]_B \cap X \neq \emptyset \right\}$$

$$\overline{B}(x) = \bigcup_{x \in \mathbb{U}} \left\{ B(x) : B(x) \cap X \neq \emptyset \right\}$$

- These *approximations* are essentially the main components from rough set theory that are utilised within R-fuzzy sets

R-Fuzzy Set

Let the pair $apr = \left(J_x, B\right)$ be an approximation space on a set of values $J_x = \{v_1, v_2, \ldots, v_n\} \subseteq [0, 1]$, and let J_x/B denote the set of all equivalence classes of B. Let $\left(\underline{M}_A(x), \overline{M}_A(x)\right)$ be a rough set in apr. An R-fuzzy set A is characterised by a rough set as its membership function $\left(\underline{M}_A(x), \overline{M}_A(x)\right)$, where $x \in \mathbb{U}$.

- An *R-fuzzy set* is given by the formal expression:

$$A = \left\{\left\langle x, \left(\underline{M}_A(x), \overline{M}_A(x)\right)\right\rangle \,\middle|\, \forall x \in \mathbb{U}, \underline{M}_A(x) \subseteq \overline{M}_A(x) \subseteq J_x\right\}$$

$$A = \sum_{x \in \mathbb{U}} \left(\underline{M_A}(x), \overline{M_A}(x)\right) / x$$

R-Fuzzy Preliminaries

- For each pair $\big((x_i), c_j\big)$ where $x_i \in \mathbb{U}$ and $c_j \in C$, a set $M_{cj}(x_i) \subseteq J_x$ is created:

$$M_{cj}(x_i) = \{v \mid v \in J_x, v \xrightarrow{\left(d(x_i), c_j\right)} \text{YES}\}$$

- The *lower approximation* of the rough set $M(x_i)$ for the membership function described by $d(x_i)$ is given by:

$$\underline{M}(x_i) = \bigcap_j M_{cj}(x_i)$$

- The *upper approximation* of the rough set $M(x_i)$ for the membership function described by $d(x_i)$ is given by:

$$\overline{M}(x_i) = \bigcup_j M_{cj}(x_i)$$

- The *rough set* approximating the membership $d(x_i)$ for x_i is given as:

$$M(x_i) = \left(\bigcap_j M_{cj}(x_i), \bigcup_j M_{cj}(x_i)\right)$$

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Significance Measure Preliminaries

Significance Measure

Using the same notations that described an R-fuzzy set, assume that an R-fuzzy set $M(x_i)$ has already been created, and that a membership set J_x and a criteria set C are also known. Given that $|N|$ is the cardinality of all generated subsets $M_{cj}(x_i)$, and that S_v is the number of subsets that contain the specified membership value being inspected. As each value $v \in J_x$ is evaluated by $c_j \in C$, the significance measure therefore counts the number of instances that v occurred over $|N|$. Making it relative to the subset of all values given by $M_{cj}(x) \subseteq J_x$.

- The *significance measure* is given by the formal expression:

$$\gamma_{\bar{A}}\{v\} = \frac{S_v}{|N|}$$

Significance Measure Preliminaries

- The significance measure expresses the *conditional probability* that $v \in J_x$ belongs to the R-fuzzy set $M(x_i)$, given by its descriptor $d(x_i)$
- The value will initially be presented as a fraction, where the denominator $|N|$ will be *indicative* of the total number of subsets
- The numerator S_v will be the number of *occurrences*, that the observed membership value was accounted for
- This fraction can be translated into a real number $\in [0, 1]$, which will be indicative of its *significance* and given by its membership function: $\gamma_{\bar{A}}\{v\} : J_x \to [0, 1]$
- If the value returned by $\gamma_{\bar{A}}\{v\} = 1$, then that particular membership value has been agreed upon by *all* in the criteria set C

Significance Measure Preliminaries

- Any membership value with a returned *significance degree of* 1, will be included within the lower approximation, and as a result it will also be included in the upper approximation:

$$\underline{M}_A = \{\gamma_{\bar{A}}\{v\} = 1 \mid v \in J_x \subseteq [0,1]\}$$

- Any membership value with a returned significance degree of *greater than* 0, will be included in just the upper approximation:

$$\overline{M}_A = \{\gamma_{\bar{A}}\{v\} > 0 \mid v \in J_x \subseteq [0,1]\}$$

- Any membership value with a returned significance degree of 0, will be completely *ignored* and not included in any approximation set

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Grey Relational Analysis Preliminaries

- We adopt the use of the *grey incidence analysis*
- Traditional grey incidence analysis is concerned with *identifying* which factors of a system are more important than others
- Establishing which factors can be identified as being *favourable* and equally, which factors are *detrimental*
- Comparing *characteristic* sequences against *behavioural* factors to ascertain how much the sequences are alike
- This information can then be used in terms of identifying if more *emphasis* should be applied to a particular behaviour or not
- We make use of the *traditional* absolute degree of grey incidence and employ it in an *untraditional* way

Grey Relational Analysis Preliminaries

- The characteristic sequences of a system $Y_1, Y_2, \ldots, Y_n$, against its behavioural factor sequences $X_1, X_2, \ldots, X_m$, all of which must be of the same *magnitude*
- $\Gamma = [\gamma_{ij}]$, where each entry in the i^{th} row of the matrix is the degree of *grey incidence* for the corresponding characteristic sequence Y_i, and relevant behavioural factors $X_1, X_2, \ldots, X_m$
- Each entry for the j^{th} column is reference to the degrees of *grey incidence* for the characteristic sequences $Y_1, Y_2, \ldots, Y_n$ and behavioural factors X_m
- There are several *variations* of the degree of incidence but we are only concerned with...

Degree of Grey Incidence

Absolute degree of grey incidence

Assume that X_i and $X_j \in \mathbb{U}$ are two sequences of data with the same magnitude, that are defined as the sum of the distances between two consecutive time points, whose zero starting points have already been computed:

$$s_i = \int_1^n (X_i - x_i(1))dt$$

$$s_i - s_j = \int_1^n (X_i^0 - X_j^0)dt$$

$$\epsilon_{ij} = \frac{1 + |s_i| + |s_j|}{1 + |s_i| + |s_j| + |s_i - s_j|}$$

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A Worked Example - Perception

- Assume that $F = \{f_1, f_2, \ldots, f_9\}$ is a set containing 9 different colour swatches, all of which are a *variation* of the colour red:

$$
\begin{aligned}
f_1 &\rightarrow [204, 0, 0] \rightarrow \\
f_2 &\rightarrow [153, 0, 0] \rightarrow \\
f_3 &\rightarrow [255, 102, 102] \rightarrow \\
f_4 &\rightarrow [51, 0, 0] \rightarrow \\
f_5 &\rightarrow [255, 153, 153] \rightarrow \\
f_6 &\rightarrow [102, 0, 0] \rightarrow \\
f_7 &\rightarrow [255, 204, 204] \rightarrow \\
f_8 &\rightarrow [255, 0, 0] \rightarrow \\
f_9 &\rightarrow [255, 51, 51] \rightarrow
\end{aligned}
$$

- The *average* RGB value from each swatch is taken and given as:

$$N = \{68, 51, 153, 17, 187, 34, 221, 85, 119\}$$

A Worked Example - Perception

- Based on the previous steps, one can now derive a *fuzzy membership set* using a simple linear function:

$$\mu(f_i) = \frac{N_i - N_{\min}}{N_{\max} - N_{\min}}$$

- The *fuzzy membership set* is given as:

$$J_x = \{0.25, 0.17, 0.67, 0.00, 0.83, 0.08, 1.00, 0.33, 0.50\}$$

- Assume that the criteria set $C = \{p_1, p_2, \ldots, p_{15}\}$ contains the perceptions of 15 individuals.
- All of whom have given their *perceived perception* for each of the swatches by using one of 3 possible descriptors:

$LR \rightarrow$ Light Red $\quad$ $R \rightarrow$ Red $\quad$ $DR \rightarrow$ Dark Red

A Worked Example - Perception

#	Age	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9
p_1	20	R	DR	LR	DR	LR	DR	LR	R	LR
p_2	30	R	DR	LR	DR	LR	DR	LR	R	R
p_3	20	R	DR	LR	DR	LR	DR	LR	R	LR
p_4	25	R	DR	LR	DR	LR	DR	LR	R	LR
p_5	25	R	DR	LR	DR	LR	DR	LR	R	LR
p_6	20	R	DR	LR	DR	LR	DR	LR	R	LR
p_7	20	R	DR	LR	DR	LR	DR	LR	R	LR
p_8	25	R	DR	LR	DR	LR	DR	LR	R	R
p_9	25	R	DR	LR	DR	LR	DR	LR	R	R
p_{10}	30	R	DR	LR	DR	LR	DR	LR	R	R
p_{11}	20	R	DR	LR	DR	LR	DR	LR	R	LR
p_{12}	25	R	DR	LR	DR	LR	DR	LR	R	LR
p_{13}	30	R	DR	LR	DR	LR	DR	LR	R	R
p_{14}	30	R	DR	LR	DR	LR	DR	LR	R	R
p_{15}	30	R	DR	LR	DR	LR	DR	LR	R	R

Table 1: The collected perceptions

A Worked Example - Perception

- The *final* generated R-fuzzy sets based on the collected subsets for *LR*, *R* and *DR*, respectively, are given as:

$$LR = (\{0.67, 0.83, 1.00\}, \{0.50, 0.67, 0.83, 1.00\})$$
$$R = (\{0.25, 0.33\}, \{0.25, 0.33, 0.50\})$$
$$DR = (\{0.00, 0.08, 0.17\}, \{0.00, 0.08, 0.17\})$$

- The returned degrees of *significance* can be seen in the following table...

LR		*R*		*DR*	
J_x	γ	J_x	γ	J_x	γ
$\gamma_{\overline{LR}}\{0.00\} =$	0.00	$\gamma_{\overline{R}}\{0.00\} =$	0.00	$\gamma_{\overline{DR}}\{0.00\} =$	1.00
$\gamma_{\overline{LR}}\{0.08\} =$	0.00	$\gamma_{\overline{R}}\{0.08\} =$	0.00	$\gamma_{\overline{DR}}\{0.08\} =$	1.00
$\gamma_{\overline{LR}}\{0.17\} =$	0.00	$\gamma_{\overline{R}}\{0.17\} =$	0.00	$\gamma_{\overline{DR}}\{0.17\} =$	1.00
$\gamma_{\overline{LR}}\{0.25\} =$	0.00	$\gamma_{\overline{R}}\{0.25\} =$	1.00	$\gamma_{\overline{DR}}\{0.25\} =$	0.00
$\gamma_{\overline{LR}}\{0.33\} =$	0.00	$\gamma_{\overline{R}}\{0.33\} =$	1.00	$\gamma_{\overline{DR}}\{0.33\} =$	0.00
$\gamma_{\overline{LR}}\{0.50\} =$	0.53	$\gamma_{\overline{R}}\{0.50\} =$	0.47	$\gamma_{\overline{DR}}\{0.50\} =$	0.00
$\gamma_{\overline{LR}}\{0.67\} =$	1.00	$\gamma_{\overline{R}}\{0.67\} =$	0.00	$\gamma_{\overline{DR}}\{0.67\} =$	0.00
$\gamma_{\overline{LR}}\{0.83\} =$	1.00	$\gamma_{\overline{R}}\{0.83\} =$	0.00	$\gamma_{\overline{DR}}\{0.83\} =$	0.00
$\gamma_{\overline{LR}}\{1.00\} =$	1.00	$\gamma_{\overline{R}}\{1.00\} =$	0.00	$\gamma_{\overline{DR}}\{1.00\} =$	0.00

Table 2: The degrees of significance for the entire populous

LR		R		DR	
J_x	γ	J_x	γ	J_x	γ
$\gamma \overline{LR}\{0.00\} =$	0.00	$\gamma \overline{R}\{0.00\} =$	0.00	$\gamma \overline{DR}\{0.00\} =$	1.00
$\gamma \overline{LR}\{0.08\} =$	0.00	$\gamma \overline{R}\{0.08\} =$	0.00	$\gamma \overline{DR}\{0.08\} =$	1.00
$\gamma \overline{LR}\{0.17\} =$	0.00	$\gamma \overline{R}\{0.17\} =$	0.00	$\gamma \overline{DR}\{0.17\} =$	1.00
$\gamma \overline{LR}\{0.25\} =$	0.00	$\gamma \overline{R}\{0.25\} =$	1.00	$\gamma \overline{DR}\{0.25\} =$	0.00
$\gamma \overline{LR}\{0.33\} =$	0.00	$\gamma \overline{R}\{0.33\} =$	1.00	$\gamma \overline{DR}\{0.33\} =$	0.00
$\gamma \overline{LR}\{0.50\} =$	1.00	$\gamma \overline{R}\{0.50\} =$	0.00	$\gamma \overline{DR}\{0.50\} =$	0.00
$\gamma \overline{LR}\{0.67\} =$	1.00	$\gamma \overline{R}\{0.67\} =$	0.00	$\gamma \overline{DR}\{0.67\} =$	0.00
$\gamma \overline{LR}\{0.83\} =$	1.00	$\gamma \overline{R}\{0.83\} =$	0.00	$\gamma \overline{DR}\{0.83\} =$	0.00
$\gamma \overline{LR}\{1.00\} =$	1.00	$\gamma \overline{R}\{1.00\} =$	0.00	$\gamma \overline{DR}\{1.00\} =$	0.00

Table 3: The degrees of significance for - 20 year olds

LR		*R*		*DR*	
J_x	γ	J_x	γ	J_x	γ
$\gamma_{\overline{LR}}\{0.00\} =$	0.00	$\gamma_{\overline{R}}\{0.00\} =$	0.00	$\gamma_{\overline{DR}}\{0.00\} =$	1.00
$\gamma_{\overline{LR}}\{0.08\} =$	0.00	$\gamma_{\overline{R}}\{0.08\} =$	0.00	$\gamma_{\overline{DR}}\{0.08\} =$	1.00
$\gamma_{\overline{LR}}\{0.17\} =$	0.00	$\gamma_{\overline{R}}\{0.17\} =$	0.00	$\gamma_{\overline{DR}}\{0.17\} =$	1.00
$\gamma_{\overline{LR}}\{0.25\} =$	0.00	$\gamma_{\overline{R}}\{0.25\} =$	1.00	$\gamma_{\overline{DR}}\{0.25\} =$	0.00
$\gamma_{\overline{LR}}\{0.33\} =$	0.00	$\gamma_{\overline{R}}\{0.33\} =$	1.00	$\gamma_{\overline{DR}}\{0.33\} =$	0.00
$\gamma_{\overline{LR}}\{0.50\} =$	0.60	$\gamma_{\overline{R}}\{0.50\} =$	0.40	$\gamma_{\overline{DR}}\{0.50\} =$	0.00
$\gamma_{\overline{LR}}\{0.67\} =$	1.00	$\gamma_{\overline{R}}\{0.67\} =$	0.00	$\gamma_{\overline{DR}}\{0.67\} =$	0.00
$\gamma_{\overline{LR}}\{0.83\} =$	1.00	$\gamma_{\overline{R}}\{0.83\} =$	0.00	$\gamma_{\overline{DR}}\{0.83\} =$	0.00
$\gamma_{\overline{LR}}\{1.00\} =$	1.00	$\gamma_{\overline{R}}\{1.00\} =$	0.00	$\gamma_{\overline{DR}}\{1.00\} =$	0.00

Table 4: The degrees of significance for - 25 year olds

LR		R		DR	
J_x	γ	J_x	γ	J_x	γ
$\gamma \frac{}{LR}\{0.00\} =$	0.00	$\gamma \frac{}{R}\{0.00\} =$	0.00	$\gamma \frac{}{DR}\{0.00\} =$	1.00
$\gamma \frac{}{LR}\{0.08\} =$	0.00	$\gamma \frac{}{R}\{0.08\} =$	0.00	$\gamma \frac{}{DR}\{0.08\} =$	1.00
$\gamma \frac{}{LR}\{0.17\} =$	0.00	$\gamma \frac{}{R}\{0.17\} =$	0.00	$\gamma \frac{}{DR}\{0.17\} =$	1.00
$\gamma \frac{}{LR}\{0.25\} =$	0.00	$\gamma \frac{}{R}\{0.25\} =$	1.00	$\gamma \frac{}{DR}\{0.25\} =$	0.00
$\gamma \frac{}{LR}\{0.33\} =$	0.00	$\gamma \frac{}{R}\{0.33\} =$	1.00	$\gamma \frac{}{DR}\{0.33\} =$	0.00
$\gamma \frac{}{LR}\{0.50\} =$	0.00	$\gamma \frac{}{R}\{0.50\} =$	1.00	$\gamma \frac{}{DR}\{0.50\} =$	0.00
$\gamma \frac{}{LR}\{0.67\} =$	1.00	$\gamma \frac{}{R}\{0.67\} =$	0.00	$\gamma \frac{}{DR}\{0.67\} =$	0.00
$\gamma \frac{}{LR}\{0.83\} =$	1.00	$\gamma \frac{}{R}\{0.83\} =$	0.00	$\gamma \frac{}{DR}\{0.83\} =$	0.00
$\gamma \frac{}{LR}\{1.00\} =$	1.00	$\gamma \frac{}{R}\{1.00\} =$	0.00	$\gamma \frac{}{DR}\{1.00\} =$	0.00

Table 5: The degrees of significance for - 30 year olds

A Worked Example - Perception

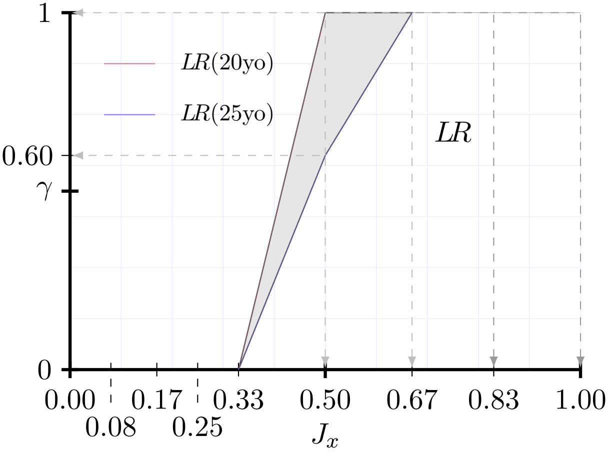

Figure 1: The comparability between two *LR* R-fuzzy sets, one generated for the age cluster 20 year olds, the other, 25 year olds

A Worked Example - Perception

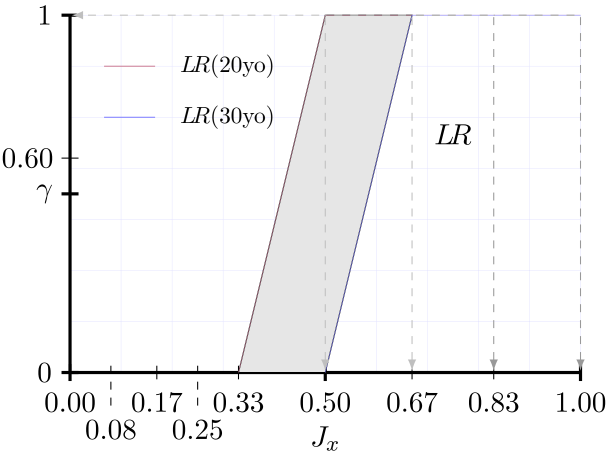

Figure 2: The comparability between two *LR* R-fuzzy sets, one generated for the age cluster 20 year olds, the other, 30 year olds

A Worked Example - Perception

LR	20yo	25yo	30yo	*R*	20yo	25yo	30yo	*DR*	20yo	25yo	30yo
20yo	$\epsilon(1.00)$	$\epsilon(0.950)$	$\epsilon(0.875)$	20yo	$\epsilon(1.00)$	$\epsilon(0.931)$	$\epsilon(0.857)$	20yo	$\epsilon(1.00)$	$\epsilon(1.00)$	$\epsilon(1.00)$
25yo	-	$\epsilon(1.00)$	$\epsilon(0.916)$	25yo	-	$\epsilon(1.00)$	$\epsilon(0.914)$	25yo	-	$\epsilon(1.00)$	$\epsilon(1.00)$
30yo	-	-	$\epsilon(1.00)$	30yo	-	-	$\epsilon(1.00)$	30yo	-	-	$\epsilon(1.00)$

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Final Remarks

- An R-fuzzy approach allows for both the *general* collective consensus and *individual* perspectives to be encapsulated
- The significance measure *quantifies* the values contained within an R-fuzzy set
- It provides a means to understand the *strength* and *weakness* of any contained value
- Each generated R-fuzzy set and corresponding significance measure can be seen as a *sequence* of discretised points
- If the data contains *clusters* of cohorts, isolated sub R-fuzzy sets can be further generated
- The use of the *absolute degree of grey incidence* can then be used to compute the difference between the metric spaces
- Providing a *metric* value which can then be inferenced

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