Leverhulme Trust Grey Systems and Its Applications

# Discrete Grey Forecast Modelling: Principle, Models and Further Studies

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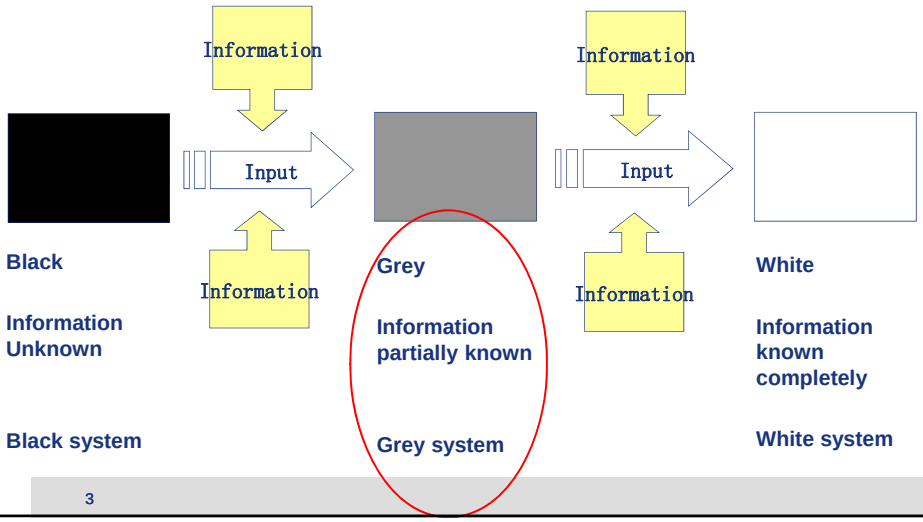
## Contents

## Introduction

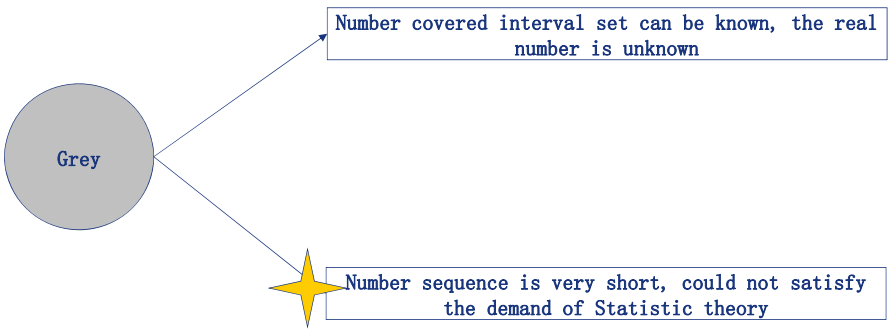
"Grey system", Deng Julong, 1982








## Introduction

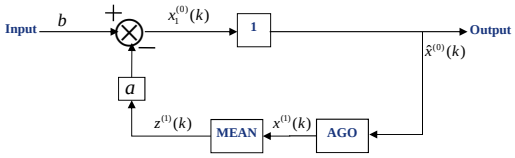


## Introduction

"Grey forecasting",1983

"Grey forecast model", 1984

- *GM (1, 1)*,single-variable grey dynamic model
- *GM (1, N)*, multi-variable grey dynamic model




GM(1, 1) Model

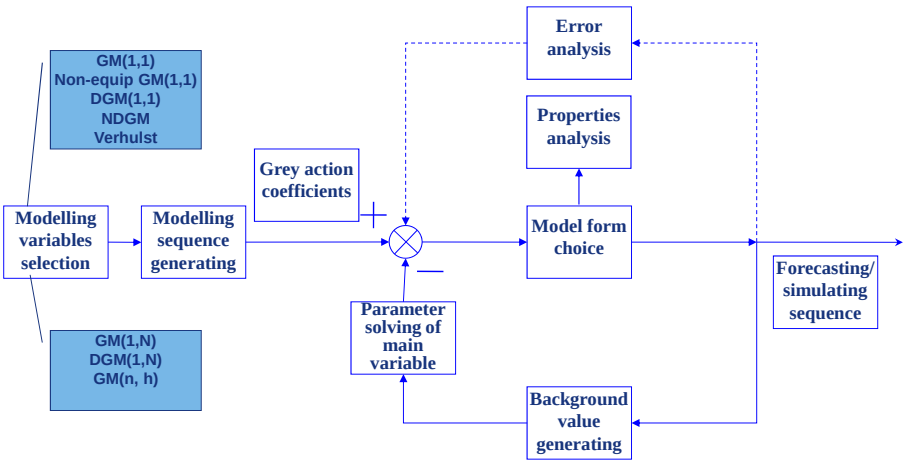
$$\frac{dx^{(1)}}{dt} + ax^{(1)} = b$$

$$x^{(0)}(k) + az^{(1)}(k) = b$$





## Principle of grey forecast modelling



| Country or Region | Country or Region | Country or Region | Country or Region | Country or Region | Country or Region |
|---|---|---|---|---|---|
| AUSTRALIA | ENGLAND | IRELAND | NIGERIA | SCOTLAND | TAIWAN |
| BELGIUM | FINLAND | ITALY | NORWAY | SENEGAL | THAILAND |
| BRAZIL | FRANCE | JAPAN | PEOPLES R CHINA | SERBIA | TURKEY |
| CANADA | GERMANY | LATVIA | POLAND | SINGAPORE | U ARAB EMIRATES |
| CHILE | GREECE | MALAYSIA | PORTUGAL | SOUTH KOREA | USA |
| COLOMBIA | HUNGARY | MEXICO | QATAR | SPAIN | VIETNAM |
| CYPRUS | INDIA | NETHERLANDS | ROMANIA | SWEDEN | WALES |
| DENMARK | IRAN | NEW ZEALAND | RUSSIA | SWITZERLAND | |





# Contents

## Theoretical breakthroughs of GFMs

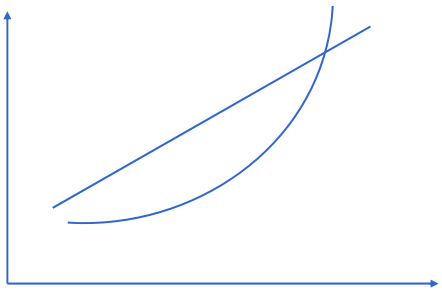
Up to present, GM(1,1) model is the most famous forecasting model in grey system theory and it has been used in various fields.

Continuous equation

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = b$$

Discrete equation

$$x^{(0)}(k) + az^{(1)}(k) = b$$





## Theoretical breakthroughs of GFMs

- Literatures shows that GM(1,1) model can produce high precision in a large amount cases, but sometime the results is not very good.
- What's the problem?
- In our opinion, the main point is that in the process of creating a GM (1, 1) model, estimating parameters adopted a discrete equation while calculating simulative and predictive values adopted a continuous equation. The difference between the two equations resulted in low simulative and forecasting precision.
- The discrete grey forecasting model (DGM) is constructed to solve such problem.
- In DGM, the accumulating generation of original sequence is keep. The parameter estimation and simulative value are calculated with the same equation.

## Theoretical breakthroughs of GFMs

**Definition 1** Assume that the sequence

$$X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(n)\}$$

is an original data sequence, the sequence

$$X^{(1)} = \{x^{(1)}(1), x^{(1)}(2), \cdots, x^{(1)}(n)\}$$

is the accumulated generation sequence of $X^{(0)}$, where

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i), k = 1, 2, \cdots, n$$

The equation

$$x^{(1)}(k+1) = \beta_1 x^{(1)}(k) + \beta_2$$

Single variable

is called discrete grey model abbreviated as DGM（1，1）





## Theoretical breakthroughs of GFMs

### Time response equation of GM(1,1) model

$$\hat{x}^{(1)}(k+1) = (x^{(0)}(1) - \frac{b}{a})e^{-ak} + \frac{b}{a}, k = 1, 2, \cdots, n-1$$

### Recursive function of DGM model

$$\begin{cases} \hat{x}^{(1)}(k+1) = \beta_1^k (\hat{x}^{(1)}(1) - \frac{\beta_2}{1-\beta_1}) + \frac{\beta_2}{1-\beta_1}, k = 1, 2, \cdots, n-1 \\ \hat{x}^{(1)}(1) = x^{(0)}(1) + \beta_3 \end{cases}$$

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## Theoretical breakthroughs of GFMs

Therefore, when the value of $a$ is less, $\beta_1 \approx e^{-a}$, substitute $e^{-a}$ with $\beta_1$ ,then the result of GM(1,1) model is infinite approximate to the result of DGM model. Therefore, DGM model and GM(1,1) model can be consider as the different expressions of the same model. When the value of $a$ is less, DGM model and GM(1,1) model can substitute each other.

when the value of  is not less, GM(1,1) model and DGM model are different.

**Xie Naiming, Liu Sifeng, Discrete grey forecasting model and its optimization, Applied Mathematical Modelling, 2009,Vol.33,No.1: 1173-1186**

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## Theoretical breakthroughs of GFMs

- **Xie Nai-ming, Liu Si-feng, Yang Ying-jie, Yuan Chao-qing, On novel grey forecasting model based on non-homogeneous index sequence, Applied Mathematical Modelling, 2013, Vol.37, No.7: 5059-5068**
- **Xie Nai-ming, Zhu Chao-yu, Liu Si-feng, Yang Ying-jie,On discrete grey system forecasting model corresponding with polynomial time-vary sequence, Journal of Grey System, 2013,Vol.25,No.4: 1-18.**
- **Xie Naiming, Zhu Chaoyu, Zheng Jing, Expansion modelling of discrete grey model based on multi-factor information aggregation, Journal of Systems Engineering and Electronics, 2014, Vol.25,No.5: 833-839.**

**Single variable grey forecasting model DGM(1, 1)** → **multi-variables** → **Grey model of multi-variables**

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## Grey model of multi-variables

GM (n, h) model

$$\frac{d^n x_1^{(1)}}{dt^n}+a_1\frac{d^{n-1}x_1^{(1)}}{dt^{n-1}}+a_2\frac{d^{n-2}x_1^{(1)}}{dt^{n-2}}+\cdots+a_{n-1}\frac{dx_1^{(1)}}{dt}+a_n x_1^{(1)}=b_1x_2^{(1)}+b_2x_3^{(1)}+\cdots+b_{h-1}x_h^{(1)}+b_h$$

$$\alpha^{(n)}x_1^{(1)}(k)+\sum_{i=1}^{n-1}a_i\alpha^{(n-i)}x_1^{(1)}(k)+a_n z_1^{(1)}(k)=\sum_{j=1}^{h-1}b_j x_{j+1}^{(1)}(k)+b_h$$

Multi-variable DGM model

$$x_1^{(1)}(k)+\alpha_1x_1^{(1)}(k-1)+\alpha_2x_1^{(1)}(k-2)+\cdots+\alpha_n x_1^{(1)}(k-n)=\sum_{i=1}^{h-1}\beta_j x_{j+1}^{(1)}(k)+\beta_h$$

Similar with GM(1,1) and single variable DGM model, GM(n, h) model and multi-variable DGM model are equal to each other through data transformation.





## Case A: Mobile telecommunication costumer number forecasting-DGM(1,N)

Recently, Mobile telecommunication rapidly increased in China. The mobile company need to forecast customer amount to arrange devices and make service strategy.

Current models mainly focus on single variable forecasting while did not consider influent factors.

| Year | Customer amount (万户) $X_1$ | Mobile device amount(万户) $X_2$ | GDP(亿元) $X_3$ | Population(万人) $X_4$ | Long distance device（万端） $X_5$ | Local device(万门) $X_6$ |
|---|---|---|---|---|---|---|
| 2000 | 8453.3 | 13985.6 | 99214.6 | 126743 | 563.5498 | 17825.6 |
| 2001 | 14522.2 | 21926.3 | 109655.2 | 127627 | 703.5769 | 25566.3 |
| 2002 | 20600.5 | 27400.3 | 120332.7 | 128453 | 773.01 | 28656.8 |
| 2003 | 26995.3 | 33698.4 | 135822.8 | 129227 | 869.3998 | 35082.5 |
| 2004 | 33482.4 | 39684.3 | 159878.3 | 129988 | 1262.998 | 42346.9 |
| 2005 | 39340.6 | 48241.7 | 184937.4 | 130756 | 1371.631 | 47196.1 |
| 2006 | 46105.8 | 61032 | 216314.4 | 131448 | 1442.343 | 50279.9 |
| 2007 | 54730.6 | 85496.1 | 265810.3 | 132129 | 1709.221 | 51034.6 |
| 2008 | 64124.5 | 114531.4 | 314045.4 | 132802 | 1690.719 | 50863.2 |
| 2009 | 74721.4 | 144084.7 | 340902.8 | 133450 | 1684.903 | 49265.6 |
| 2010 | 85900.3 | 150284.9 | 401202.0 | 134091 | 1641.464 | 46537.3 |

## Case A: Mobile telecommunication costumer number forecasting-DGM(1,N)

We construct Grey relational models between X1 and other variables. As show in the table, X2 and X3 are strongly related with X1 while other 3 variables are not.

2000-2004, Mobile device amount mainly drive the increasing trend.

While after 2005, the GDP became the main drive variable.

| 年份 | Mobile device amount(万户) $X_2$ | GDP(亿元) $X_3$ | Population(万人) $X_4$ | Long distance device（万端） $X_5$ | Local device(万门) $X_6$ |
|---|---|---|---|---|---|
| 2000-2004 | **0.80** | 0.61 | 0.58 | 0.65 | 0.69 |
| 2005-2009 | 0.64 | **0.94** | 0.64 | 0.72 | 0.67 |



## Case A: Mobile telecommunication costumer number forecasting-DGM(1,N)

# DGM(1, N)

$$x_1^{(1)}(k) - 0.63392x_1^{(1)}(k-1) = -0.019643x_2^{(1)}(k) + 0.10377x_3^{(1)}(k) - 3350$$

## Case A: Mobile telecommunication costumer number forecasting-DGM(1,N)

| Year | Amount | DGM(1,1) | | DGM(1,N) | | Linear Regression | | GM(1,N) | |
|---|---|---|---|---|---|---|---|---|---|
| | | Value | Error (%) | Value | Error (%) | Value | Error (%) | Value | Error (%) |
| 2000 | 8453.3 | 8453.3 | 0 | 8453.3 | 0 | 14587 | 72.6 | 8453.3 | 0 |
| 2001 | 14522.2 | 18914 | 30.2 | 14524 | 0.1 | 16937 | 16.6 | 14983.09 | 3.2 |
| 2002 | 20600.5 | 22565 | 9.5 | 21155 | 2.7 | 19555 | 5.1 | 28066.44 | 36.2 |
| 2003 | 26995.3 | 26920 | 0.3 | 26842 | 0.6 | 23484 | 13.0 | 38682.16 | 43.3 |
| 2004 | 33482.4 | 32115 | 4.1 | 32826 | 2 | 29893 | 10.7 | 49428.24 | 47.6 |
| 2005 | 39340.6 | 38314 | 2.6 | 39052 | 0.7 | 36382 | 7.5 | 59449.04 | 51.1 |
| 2006 | 46105.8 | 45708 | 0.9 | 46003 | 0.3 | 44340 | 3.8 | 70129.5 | 52.1 |
| 2007 | 54730.6 | 54530 | 0.4 | 55065 | 0.6 | 56546 | 3.3 | 84623.3 | 54.6 |
| 2008 | 64124.5 | 65055 | 1.5 | 65244 | 1.8 | 68022 | 6.1 | 98316.4 | 53.3 |
| 2009 | 74721.4 | 77610 | 3.9 | 73904 | 1.1 | 73331 | 1.9 | 104952.9 | 40.5 |
| 2010 | 85900.3 | 92589 | 7.8 | 85523 | 0.5 | 90107 | 4.9 | 124507.1 | 44.9 |



## GM model based on interval grey sequence

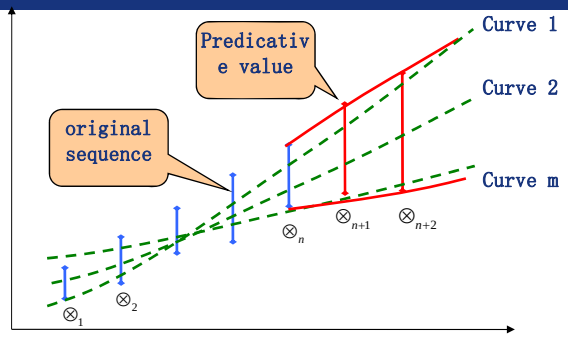
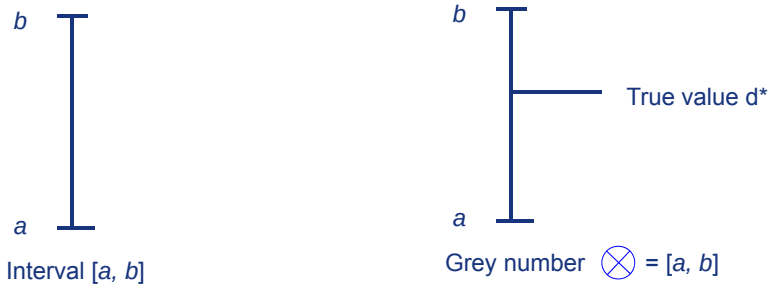



Fig. Simulative and predicative value of grey number sequence

Definition of grey numbers → Operations of grey numbers → Simulative and predicative value based on optimized methods

## GM model based on interval grey sequence

**Definition 2** The whole character information of an object is called proposition-information field or information background of proposition which is marked as $\mathscr{P}(\theta)$








## GM model based on interval grey sequence

| | Assumed Real number | Grey numbers | Information background | Grey degree |
|---|---|---|---|---|
| A | 3 | [2,4] | [0,10] | 0.2 |
| B | 5 | [2,8] | [0,20] | 0.3 |
| A+B=C | 8 | [4,12] | [0,30] | 0.27 |

| | | |
|---|---|---|
| C-A | 5 | [0, 10] |
| =B | =B | ≠B |

**A, B: Simple grey number**

**C: Integrated grey number**

## GM model based on interval grey sequence

**Definition 3** Due to some proposition $\mathscr{F}$, $\mathscr{F}(\theta)$ is the proposition-information field. For proposition is expressed incompletely or people is difficult to catch all the information of the proposition we can merely know the probable range of the proposition's value or a set including several values. Marked the set as $D$. So the proposition can be expressed as an uncertainty number $\otimes$ under the $\mathscr{F}(\theta)$. $d^*$ is the true value of the proposition. Then we call

(1) $\otimes$ is a grey number under the proposition $\mathscr{F}$.

(2) $D$ is the value-covered set of $\otimes$.





## GM model based on interval grey sequence

**Definition 4** Let $D_i$ and $d_i^*$ as the value-covered set and true value of grey number $\otimes_i$. Let $D_j$ and $d_j^*$ as the value-covered set and true value of grey number $\otimes_j$. $\circ$ as an operation. Let $\otimes_{ij}$ as the result of $\otimes_i$ and $\otimes_j$ on the $\circ$ operation. Let $D_{ij}$ as the value-covered of the grey number $\otimes_{ij}$. Then we have the general $\circ$ operation formula.

$$\otimes_i \circ \otimes_j = \otimes_{ij} \Leftrightarrow \left\{\forall d_i^* \in D_i, d_j^* \in D_j, \exists d_{ij}^* \in D_{ij} \Rightarrow d_i^* \circ d_j^* = d_{ij}^*, D_i \circ D_j = D_{ij}\right\}$$

abbreviated as $\otimes_i \circ \otimes_j = \otimes_{ij}$.

Due to the grey numbers $\otimes_i$ and $\otimes_j$, their true values $d_i^*$ and $d_j^*$ are unknown. So the most important operation is the operation of $D_i$ and $D_j$ in the general $\circ$ operation formula.

## GM model based on interval grey sequence

**Definition 5** Suppose grey numbers $\otimes_1, \otimes_2, \cdots, \otimes_n$ have the corresponding continuous covered sets $D_i = [a_i, b_i]$, if $\otimes = f(\otimes_1, \otimes_2, \cdots, \otimes_n, \circ)$ is the result under two or more $\circ$ operations. $\circ \in \{+, -, \times, \div\}$. Then the corresponding value-covered set $D = f(D_1, D_2, \cdots, D_n, \circ)$ of complex number $\otimes$ can be calculated by two optimized model:

$$\min f(x_1, x_2, \cdots, x_n, \circ) \quad s.t \begin{cases} a_1 \le x_1 \le b_1 \\ a_2 \le x_2 \le b_2 \\ \vdots \\ a_n \le x_n \le b_n \end{cases} \quad \text{and} \quad \max f(x_1, x_2, \cdots, x_n, \circ) \quad s.t \begin{cases} a_1 \le x_1 \le b_1 \\ a_2 \le x_2 \le b_2 \\ \vdots \\ a_n \le x_n \le b_n \end{cases}$$

Let $a = \min f(x_1, x_2, \cdots, x_n, \circ)$, $b = \max f(x_1, x_2, \cdots, x_n, \circ)$. Then the corresponding value-covered set $D = f(D_1, D_2, \cdots, D_n, \circ) = [a, b]$.





## GM model based on interval grey sequence

**Definition 6** Assume that the sequence

$$X^{(0)}(\otimes) = \{x^{(0)}(\otimes_1), x^{(0)}(\otimes_2), \cdots, x^{(0)}(\otimes_n)\}$$

is original grey number sequence, $x^{(0)}(\otimes_i)(i = 1, 2, \cdots, n)$ is interval grey number. Define $x^{(0)}(\otimes_1) \in [a_1, b_1], x^{(0)}(\otimes_2) \in [a_2, b_2], \cdots, x^{(0)}(\otimes_n) \in [a_n, b_n]$

$$X^{(1)}(\otimes) = \{x^{(1)}(\otimes_1), x^{(1)}(\otimes_2), \cdots, x^{(1)}(\otimes_n)\}$$

is the accumulated generation sequence of $X^{(0)}(\otimes)$, where

$$x^{(1)}(\otimes_k) = \sum_{i=1}^{k} x^{(0)}(\otimes_i), k = 1, 2, \cdots, n$$

Then the equation

$$\begin{cases} \hat{x}^{(1)}(\otimes_{k+1}) = \beta_1 \hat{x}^{(1)}(\otimes_k) + \beta_2 \\ \hat{x}^{(1)}(\otimes_1) = x^{(1)}(\otimes_1) + \beta_3 \end{cases}$$

is called interval grey number sequence discrete grey model abbreviated as IG-DGM.

## GM model based on interval grey sequence

**Theorem 1** According to the least squares estimate sequence

$$\hat{\beta} = [\beta_1, \beta_2] = (B^T B)^{-1} B^T Y$$

Set $\hat{x}^{(1)}(\otimes_1) = x^{(1)}(\otimes_1) + \beta_3$, then recursive function is given by

$$\hat{x}^{(1)}(\otimes_{k+1}) = \beta_1^k \hat{x}^{(1)}(\otimes_1) + \frac{1-\beta_1^k}{1-\beta_1} \cdot \beta_2, k = 1, 2, \cdots, n-1$$

or

$$\hat{x}^{(1)}(\otimes_{k+1}) = \beta_1^k (\hat{x}^{(1)}(\otimes_1) - \frac{\beta_2}{1-\beta_1}) + \frac{\beta_2}{1-\beta_1}, k = 1, 2, \cdots, n-1$$



## GM model based on interval grey sequence

**Theorem 2** The restored values of $\hat{x}^{(0)}(\otimes_k)$ can be given by

$$\hat{x}^{(0)}(\otimes_{k+1}) = \alpha^{(1)} \hat{x}^{(1)}(\otimes_{k+1}) = \hat{x}^{(1)}(\otimes_{k+1}) - \hat{x}^{(1)}(\otimes_k), k = 1, 2, \cdots, n-1.$$

Substitute $\hat{x}^{(1)}(\otimes_{k+1})$ and $\hat{x}^{(1)}(\otimes_k)$, we can get

$$\hat{x}^{(0)}(\otimes_{k+1}) = \hat{x}^{(1)}(\otimes_{k+1}) - \hat{x}^{(1)}(\otimes_k) = (\beta_1^k - \beta_1^{k-1})(\hat{x}^{(1)}(\otimes_1) - \frac{\beta_2}{1-\beta_1})$$

**Theorem 3**, the maximum and minimum simulate value of $\hat{x}^{(0)}(\otimes_1)$ can be get from the optimized model

$$\min \hat{x}^{(0)}(\otimes_1) = x^{(1)}(\otimes_1) + \beta_3 \quad s.t. \begin{cases} a_1 \le x^{(0)}(\otimes_1) \le b_1 \\ a_2 \le x^{(0)}(\otimes_2) \le b_2 \\ \vdots \\ a_n \le x^{(0)}(\otimes_n) \le b_n \end{cases}$$

and

$$\max \hat{x}^{(0)}(\otimes_1) = x^{(1)}(\otimes_1) + \beta_3 \quad s.t. \begin{cases} a_1 \le x^{(0)}(\otimes_1) \le b_1 \\ a_2 \le x^{(0)}(\otimes_2) \le b_2 \\ \vdots \\ a_n \le x^{(0)}(\otimes_n) \le b_n \end{cases}$$

## GM model based on interval grey sequence

**Theorem 4** The maximum and minimum simulate value of $\hat{x}^{(0)}(\otimes_i)(i=2,3,\cdots,n)$ and the forecasting value $\hat{x}^{(0)}(\otimes_i)(i=n+1,n+2,\cdots)$ can be get from the optimized model

$$\min \hat{x}^{(0)}(\otimes_i)=(\beta_1^k-\beta_1^{k-1})(x^{(1)}(\otimes_1)+\beta_3-\frac{\beta_2}{1-\beta_1})$$

$$s.t\begin{cases} a_1 \le x^{(0)}(\otimes_1) \le b_1 \\ a_2 \le x^{(0)}(\otimes_2) \le b_2 \\ \vdots \\ a_n \le x^{(0)}(\otimes_n) \le b_n \end{cases}$$

and

$$\max \hat{x}^{(0)}(\otimes_i)=(\beta_1^k-\beta_1^{k-1})(x^{(1)}(\otimes_1)+\beta_3-\frac{\beta_2}{1-\beta_1})$$

$$s.t\begin{cases} a_1 \le x^{(0)}(\otimes_1) \le b_1 \\ a_2 \le x^{(0)}(\otimes_2) \le b_2 \\ \vdots \\ a_n \le x^{(0)}(\otimes_n) \le b_n \end{cases}$$





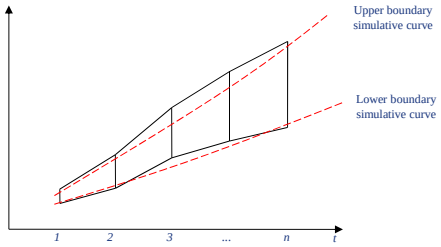
## Case B: Interval grey forecasting—IG-DGM

Numerical Example:

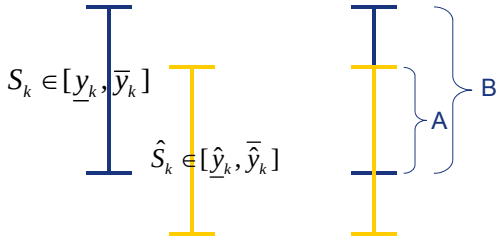
We construct Optimized IN-DGM model based on interval grey numbers sequence.

And construct DGM(1,1) model and GM(1,1) model separately with upper and lower values.

| | Upper | Lower | |
|---|---|---|---|
| 1 | 21.1 | 19.3 | [19.3, 21.1] |
| 2 | 26.5 | 24.5 | [24.5, 26.5] |
| 3 | 36.1 | 33.3 | [33.3, 36.1] |
| 4 | 52.3 | 48.7 | [48.7, 52.3] |
| 5 | 78.8 | 74.6 | [74.6, 78.8] |



## Case B: Interval grey forecasting—IG-DGM




(PSVSC(%))=1-A÷B

Percent of simulating value set covered (PSVSC(%))





## Case B: Interval grey forecasting—IG-DGM

| No. | Actual value | | | Simulate value of *IN-NDGM* model | | | *PSVSC (%)* | *APEM (%)* |
|---|---|---|---|---|---|---|---|---|
| | dL | dU | Mean | dL | dU | Mean | | |
| 1 | 21.1 | 19.3 | 20.2 | | | | | |
| 2 | 26.5 | 24.5 | 25.5 | 24.45 | 26.46 | 25.46 | 1.93 | 0.17 |
| 3 | 36.1 | 33.3 | 34.7 | 33.39 | 36.17 | 34.78 | 3.19 | 0.24 |
| 4 | 52.3 | 48.7 | 50.5 | 48.63 | 52.24 | 50.43 | 1.69 | 0.13 |
| 5 | 78.8 | 74.6 | 76.7 | 74.60 | 78.81 | 76.71 | 0.12 | 0.01 |
| *MPSVSC (%)* | | | | | | | 1.73 | |
| *MAPEM (%)* | | | | | | | | 0.14 |

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## Case B: Interval grey forecasting—IG-DGM

| No. | Simulate value of *DGM(1, 1)* model | | | | | Simulate value of *GM(1, 1)* model | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | dL | dU | Mean | PSVSC (%) | APEM (%) | dL | dU | Mean | PSVSC (%) | APEM (%) |
| 2 | 22.85 | 25.01 | 23.93 | 74.45 | 6.16 | 22.48 | 24.64 | 23.56 | 93.00 | 7.61 |
| 3 | 33.70 | 36.49 | 35.10 | 14.42 | 1.15 | 33.02 | 35.80 | 34.41 | 10.71 | 0.84 |
| 4 | 49.72 | 53.25 | 51.48 | 28.29 | 1.95 | 48.49 | 52.02 | 50.26 | 7.78 | 0.49 |
| 5 | 73.34 | 77.69 | 75.52 | 26.35 | 1.54 | 71.20 | 75.59 | 73.40 | 76.43 | 4.31 |
| *MPSVSC (%)* | | | | 35.88 | | | | | 46.98 | |
| *MAPEM (%)* | | | | | 2.70 | | | | | 3.31 |

Xie Naiming, Liu Sifeng, Yang Yingjie, Yuan Chaoqing, Grey number sequence forecasting approach for interval analysis: A case of China's Gross Domestic Product Prediction, Journal of Grey System, 2014,Vol.26, No.1: 1-14.



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## Contents

## Trends of grey forecasting models

### NO.1 Modelling sequence condition analysis

- ✓ What are the real sequences developing law for different grey forecasting models?
- ✓ Whether the errors between the real value and the simulate value of modelling sequences have a special distribution?
- ✓ How many of the data should be included in modelling process?
- ✓ How to choose the suitable sequence interval to construct the grey forecasting models?

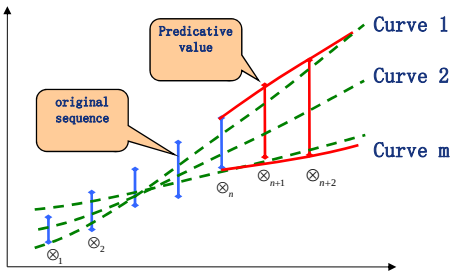




## Trends of grey forecasting models

### NO.2 Grey forecasting models based on grey information

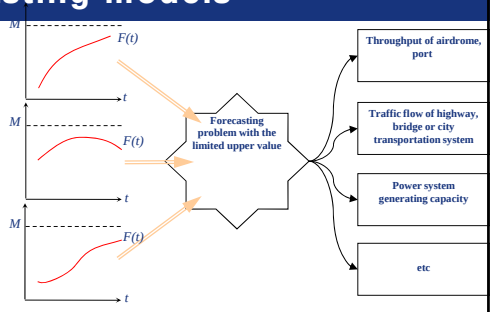
- Grey number operation
- Modelling
- Solution
- Simulation
- Test
- Real application

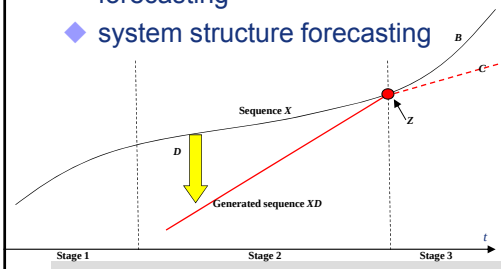


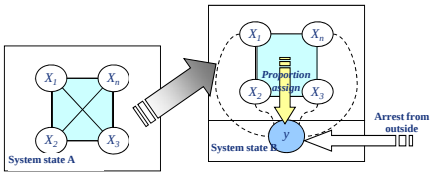
## Trends of grey forecasting models

### NO.3 Grey forecasting models for special usage

- limited upper value forecasting
- shock wave system forecasting
- system structure forecasting








## Trends of grey forecasting models

### NO.4 Compare Grey forecast models

- Compare GMs
- Compare GMs with other forecasting models(ARIMA, LR,….)

# Thank You !