



The Leverhulme Trust

# International Network on Grey Systems and its Applications

**Keynote talk:  
Machine Learning in Grey-Based Soft Computing  
techniques**

Keynote speaker: **Prof. Jose L. Salmeron**

Universidad Pablo de Olavide (Seville, Spain)

September 23, 2015

# About me

## Current position

- Prof. Jose L. Salmeron, Ph.D. CS (AI), Ph.D. BA, MSc. CS, MBA, BSc. CS, BBA  
Professor of Information Systems.  
University Pablo de Olavide at Seville (Spain)
- **Home page:** <http://eps.upo.es/salmeron/>
- **e-mail** ✉: [salmeron@acm.org](mailto:salmeron@acm.org)

## Research

- **Research interests:** Decision Making in Autonomous Systems, Fuzzy Cognitive Maps, Decision Support Systems, Forecasting, Soft computing, Computational Intelligence, Knowledge-Based Systems, and always researching in something new...
- Member of societies as ACM, Association for Logic Programming, ENBIS and others.
- **Consulting** for Getronics, KPMG, Everis, GEA21, UHK, DDTel, and others.

## 1 FCM fundamentals

- Basics
- Construction
- Analysis

## 2 Fuzzy Grey Cognitive Maps

- Grey Systems Theory
- FGCM fundamentals
- Analysis
- Example: A FGCM-based intelligent system

## 3 Machine Learning in Grey-based Soft Computing techniques

- Unsupervised Learning
- Supervised Learning

# Outline

## 1 FCM fundamentals

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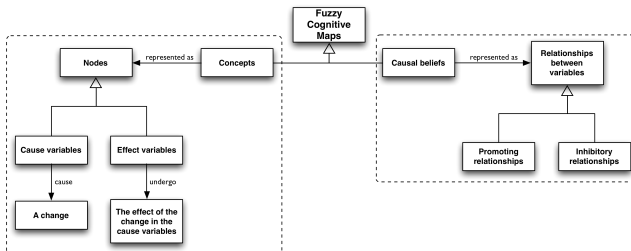
## 3 Machine Learning in Grey-based Soft Computing techniques

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# Fuzzy Cognitive Maps components

## Representation

- FCM is a soft computing technique, closely to a **one-layer recurrent dynamic neural network**.
- FCMs consist of concepts, that illustrate different aspects in the system's behavior and these concepts interact with each other showing the dynamics of the system.
- FCMs are represented by directed graphs capable of modelling relationships or casualities existing between concepts. Concepts ( $c_i$ ) are represented by nodes and edges ( $e_{ij}$ ) represent relationships between them.



# Introduction

## Definitions

A FCM can be represented as a 4-tuple

$$\Gamma = \langle \mathbf{N}, \mathbf{E}, f, r \rangle$$

where  $\mathbf{N}$  is the set of nodes,  $\mathbf{E}$  are the set of edges between nodes,  $f(\cdot)$  the activation function and  $r$  the nodes' range,  $r = \{[0, +1] \mid [-1, +1]\}$ ,  $\mathbf{N} = \{ \langle n_i \rangle \}$  where  $n_i$  are the nodes.

$\mathbf{E}$  is represented as

$$\mathbf{E} = \{ \langle e_{ij}, w_{ij} \rangle \mid n_i, n_j \in \mathbf{V} \}$$

where  $e_{ij}$  is the edge from node  $n_i$  to node  $n_j$ , and  $w_{ij}$  is the weight of the edge  $e_{ij}$ .

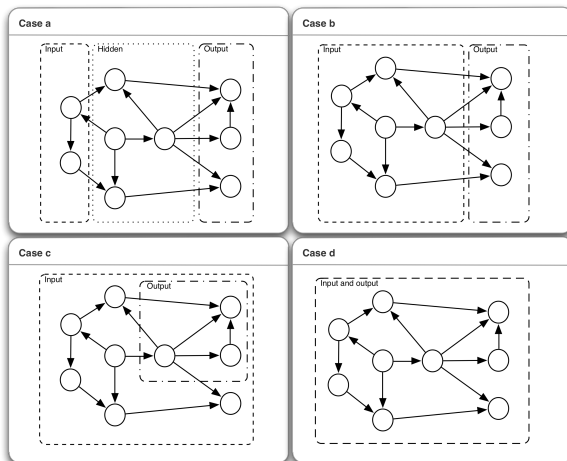
# Topology

## One-layer recurrent dynamical NN

- FCM models a system as an one-layer NN.
- Note that it is not a neural network. Figure shows just an analogy.

## Main difference with NN

- Each node has a meaning in FCM



# Expert based

## Experts issues

- Expert-based construction is strongly dependent on the experts' selection and its knowledge.

## Augmented Weighted FCM

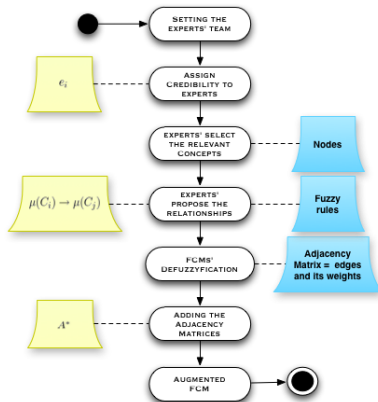
- Each expert could has a credibility weight ( $e_i$ ).

$$A^* = \sum_{k=1}^E \left( \frac{e_k}{\sum_{m=1}^E e_m} \right) \cdot A_k$$

## Defuzzification

- It is necessary to convert the fuzzy quantities into crisp quantities for further FCM processing. For instance, *Center of Gravity*:

$$COG_y = \frac{\int_{x \in X} x \cdot \mu_A(x) dx}{\int_{x \in X} \mu_A(x) dx}$$



# Delphi/Augmented FCM



S. Bueno and J.L. Salmeron (2008). Fuzzy modelling Enterprise Resource Planning tool selection. Computer Standards and Interfaces 30(3), pp. 137-147.



J.L. Salmeron (2009). Augmented Fuzzy Cognitive Maps for modelling LMS Critical Success Factors. Knowledge-Based Systems 22(4), pp. 275-278.

## Delphi method

- It is just for expert-based FCMs.
- Experts design FCM in several rounds.
- After the first one, each expert knows the overall data and he/she can adjust his/her previous judgement.

## Augmented FCM

- It doesn't need that experts change their initial judgement for consensus as Delphi methodology.
- The augmented adjacency matrix ( $A^* = [w_{ij}^*]_{m \times m}$ ) is built adding the adjacency matrix of each data source.
- If there are common nodes within the adjacency matrices, the element ( $w_{ij}^*$ ) would be  $w_{ij}^* = \frac{1}{n} \cdot \sum_{k=1}^n w_{ijk}$

# Data-driven

## Goal

- Improve the FCM model

Alg.	Learning goal	Human involvement	Data type	Activation function	Learning type
DHL	Adj. mat.	No	Single	N/A	Hebbian
BDA	Adj. mat.	No	Single	Binary	modified Hebbian
NHL	Adj. mat.	Initial	Single	Continuous	modified Hebbian
AHL	Adj. mat.	Initial	Single	Continuous	modified Hebbian
GS	Adj. mat.	No	Multiple	Continuous	Genetic
PSO	Adj. mat.	No	Multiple	Continuous	Swarm
GA	Initial vector	N/A	N/A	Continuous	Genetic
RCGA	Adj. mat.	No	Single	Continuous	Genetic

## AHL/NHL human involvement

- Initial human intervention is necessary, but later when applying the algorithm there is no human intervention needed

# Comparison

## Expert-based vs. data-driven approaches

	<b>Expert-based</b>	<b>Data-driven</b>
<i>Type of modeling</i>	<i>Deductive</i>	<i>Inductive</i>
Main objective	To create a model that is structurally understandable	To create a model that provides accurate simulations
Main application	Static analysis	Dynamic analysis
Main shortcoming	Dynamic analysis could be inaccurate	Static analysis could be inaccurate and more difficult

# Static

## Concept

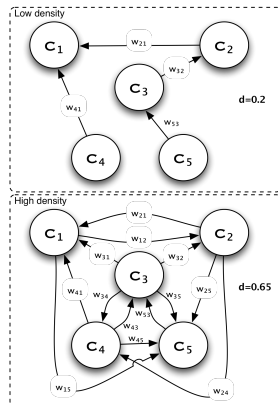
Static analysis study the characteristics of the FCM weighted directed graph that represent the model, using graph theory techniques.

## Density

Density ( $D$ ) is the relation between the nodes ( $N$ ) and the edges ( $E$ ) of the model.

$$D = \frac{E}{N \cdot (N - 1)}$$

High density indicates increased complexity in the model and respectively to the problem that the model represents.



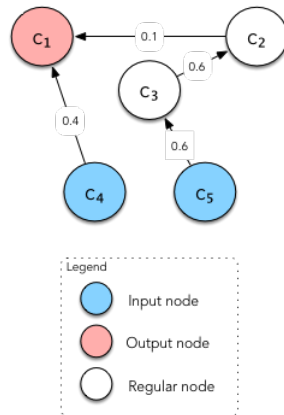
# Static

## Some measures

In-degree	$d^+(c_j) = \sum_{i=1}^n  e_{ij} $
Out-degree	$d^-(c_i) = \sum_{j=1}^n  e_{ij} $
Centrality	$c(c_i) = d^+(c_i) + d^-(c_i)$

Weighted in-degree	$d_w^+(c_j) = \sum_{i=1}^n  w_{ij} $
Weighted out-degree	$d_w^-(c_i) = \sum_{j=1}^n  w_{ij} $
Weighted centrality	$c_w(c_i) = d_w^+(c_i) + d_w^-(c_i)$

Input node	$n_i   d^+ = 0$
Output node	$n_i   d^- = 0$



# FCM Dynamics

## Concept

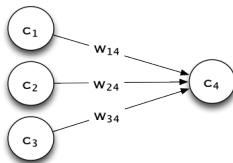
FCM is a deterministic model. It predicts future state evolution deterministically as shown in the updating rules with the expectation that the initial state vector is converged finally to a fixed point.

## Updating nodes

$$c_i(t+1) = f\left(\sum_{j=1}^n w_{ji} \cdot c_j(t)\right)$$

$$c_i(t+1) = f\left(\sum_{j=1}^n w_{ji} \cdot c_j(t) + c_i(t)\right)$$

$$c_i(t+1) = f\left(k_1 \cdot \sum_{j=1}^n w_{ji} \cdot c_j(t) + k_2 \cdot c_i(t)\right)$$



1st iteration

$$c(0) = (c_1(0) \cdots c_4(0)) \rightarrow c(1) = (f(\cdot) \cdots f(\cdot))$$

2nd iteration

$$c(1) = (c_1(1) \cdots c_4(1)) \rightarrow c(2) = (f(\cdot) \cdots f(\cdot))$$

.....

Last iteration

$$\sqrt{\sum_{i=1}^N (c_i(t) - c_i(t-1))^2} < \overbrace{\epsilon}^{\text{tolerance}}$$

# Activation functions

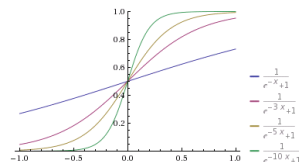


Bueno, S. and Salmeron, J.L. (2009). Benchmarking Main Activation Functions in Fuzzy Cognitive Maps. *Expert Systems with Applications* 36(3 part 1) pp. 5221-5229.

## Concept

The FCM inference process finish when the stability is reached. The FCM reaches either one of the following states following the iterations.

- Fixed-point attractor  $c(t-1) = c(t)$
- Limited cycle  $\exists t, k | (c(t-k) = c(t)) \wedge (c(t-k+1) \neq c(t))$
- Chaotic attractor



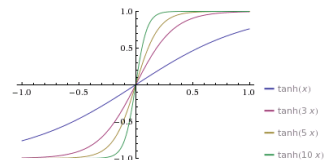
## Main activation functions

Bivalent 
$$f(c_i) = \begin{cases} 0 & \text{if } c_i \leq 0 \\ 1 & \text{if } c_i > 0 \end{cases}$$

Trivalent 
$$f(c_i) = \begin{cases} -1 & \text{if } c_i \leq -0.5 \\ 0 & \text{if } -0.5 < c_i < 0.5 \\ +1 & \text{if } c_i \geq 0.5 \end{cases}$$

Unipolar sigmoid 
$$f(c_i) = \frac{1}{1 + e^{-\lambda \cdot c_i}}$$

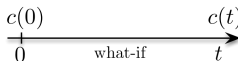
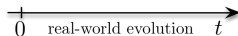
Hyperbolic tangent 
$$f(c_i) = \frac{e^{2 \cdot \lambda \cdot c_i} + 1}{e^{2 \cdot \lambda \cdot c_i} - 1}$$



# Forward-Backward

## What-if analysis

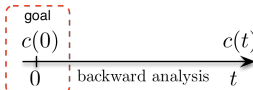
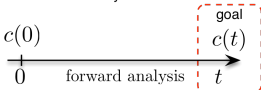
- *Goal*: Impact of  $c(0)$  over  $c(t)$
- *Method*: Running FCM dynamics



## Population-based analysis

- *Method*: Evolving a population of initial vector states  $(\{c(0)\}_{i=1}^n)$
- *Fitness*:  $\min(err)$  where  $err_i = |c(t)_i - c(t)_i^*|$
- Forward analysis
  - Finding the optimum final vector state  $(c(t))$
- Backward analysis
  - Finding  $c(0)$  that generates a specific  $c(t)$

### Population-based analysis



# Ranking FCM scenarios with TOPSIS



J.L. Salmeron, R. Vidal and A. Mena (2012). Ranking Fuzzy Cognitive Maps based scenarios with TOPSIS. Expert Systems with Applications 39(3), pp. 2443-2450.

## FCM consensus

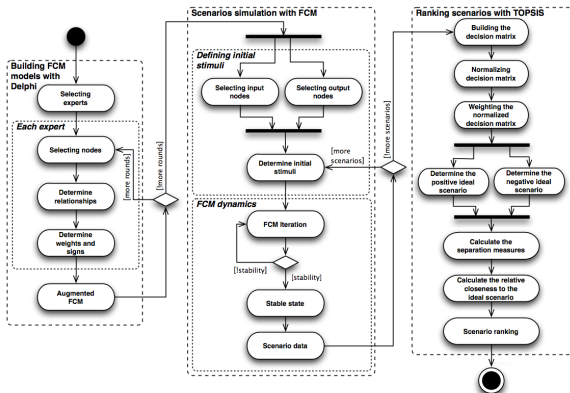
This proposal includes Delphi

## FCM dynamics

Dynamic analysis is run for each scenario

## TOPSIS-based rank

The closer scenario to the positive-ideal scenario is the best solution.



# FCM extensions

## FCM extensions

- **Fuzzy Grey Cognitive Maps**
- Rule-Based Fuzzy Cognitive Maps
- Probabilistic Fuzzy Cognitive Maps
- Intuitionistic Fuzzy Cognitive Maps
- Dynamical Cognitive Networks
- Belief Degree-Distributed Fuzzy Cognitive Maps
- Rough Cognitive Maps
- Dynamic Random Fuzzy Cognitive Maps
- Fuzzy Cognitive Networks
- Evolutionary FCMs
- Fuzzy Time Cognitive Maps
- Fuzzy Rules Incorporated with FCMs
- Timed Automata-based Fuzzy Cognitive Maps

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# Overview

## Introduction

- A very effective solving problem set of techniques within environments with high uncertainty.
- GST have been designed to analyze small data samples with poor information.

## Comparison between GST, and Fuzzy theory

- **Fuzzy theory** holds some previous information (usually based on **experience**); while **GST** deal with **objective data**, they do not need any previous information other than the data sets that need to be disposed.
- The key difference between fuzzy and grey systems concepts is **intension and extension** of the analyzed objects. While GST focus on objects with clear extension and ambiguous intension, fuzzy theory mostly analyze the objects with clear intension and ambiguous extension.

## Comparison between Black, Grey and White systems

	Black	Grey	White
Information	Unknown	Incomplete	Known
Conclusion	No result	Multiple solution	Unique solution

Expert Systems with Applications 37 (2010) 7581–7588



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journal homepage: [www.elsevier.com/locate/eswa](http://www.elsevier.com/locate/eswa)



## Modelling grey uncertainty with Fuzzy Grey Cognitive Maps

Jose L. Salmeron

*School of Engineering, University Pablo de Olavide, Ctra. de Utrera, km. 1, 41013 Seville, Spain*

# Introduction



J.L. Salmeron (2010). Modelling grey uncertainty with Fuzzy Grey Cognitive Maps. Expert Systems with Applications 37, pp. 7581-7588.

## Fuzzy Grey Cognitive Maps

- A FGCM **models unstructured knowledge through causalities through imprecise terms and grey relationships** between them based on Fuzzy Cognitive Maps.
- FGCMs are a **generalization of FCMs**, since a FGCM with all the relations' intensities represented by white numbers would be a FCM.
- FGCM **represents the human intelligence better** than FCM, because it faces unclear relations between factors and incomplete information better than FCM.

## Comparison

Requirements	Systems Dynamics	Bayesian Networks	Neural Networks	FCM	FGCM
Considers all possible connections	✓	✓	✓	✓	✓
Do not ignore the uncertainty		✓	✓	✓	✓
Directed graph with cycles	✓			✓	✓
The propagation does not carry on an established pattern	✓			✓	✓
Assume information is scarce			✓	✓	✓
Process the data source's uncertainty					✓
Consider multi-meaning problems					✓

# Introduction

## Definitions

A FGCM can be represented as a 4-tuple

$$\Psi = \langle \mathbf{N}, \mathbf{E}, f, r \rangle$$

where  $\mathbf{N}$  is the set of nodes and  $\mathbf{E}$  are the set of edges between nodes,  $f(\cdot)$  the grey activation function and  $r$  the limits nodes' state range and  $\{\underline{c}, \bar{c}\} \in \{[0, +1] | [-1, +1]\}$ . Note that  $\underline{r}$  and  $\bar{r}$  must have the same range.

As FCMs,  $\mathbf{N}$  is represented as a tuple

$$\mathbf{N} = \{ \langle n_i \rangle \}$$

where  $n_i$  are the nodes.  $\mathbf{E}$  is represented by

$$\mathbf{E} = \{ \langle e_{ij}, \otimes w_{ij} \rangle \mid n_i, n_j \in \mathbf{V} \}$$

where  $e_{ij}$  is the edge from node  $n_i$  to node  $n_j$ , and  $\otimes w_{ij}$  is the grey weight of the edge  $e_{ij}$ . Note that FGCMs include grey uncertainty within grey weights and grey nodes' states.

# Grey weights $\otimes g$ limits and measures

Grey numbers  $\otimes g \in [\underline{g}, \bar{g}]$  in FGCM

- 1  $\otimes g_1 \in [\underline{g}_1, \bar{g}_1] \mid \underline{g}_1 < \bar{g}_1$
- 2  $\otimes g_2 \in [\underline{g}_2, \bar{g}_2] \mid \underline{g}_2 < +\infty, \bar{g}_2 = +\infty$
- 3  $\otimes g_3 \in [\underline{g}_3, \bar{g}_3] \mid \underline{g}_3 = -\infty, \bar{g}_3 > -\infty$
- 4  $\otimes g_4 \in [\underline{g}_4, \bar{g}_4] \mid \underline{g}_4 = -\infty, \bar{g}_4 = +\infty$
- 5  $\otimes g_5 \in [\underline{g}_5, \bar{g}_5] \mid \underline{g}_5 = \bar{g}_5$

## Whitenization

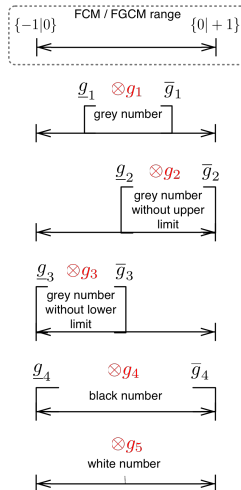
The transformation process of grey numbers in white ones

$$\hat{\otimes} g = \delta \cdot \underline{g} + (1 - \delta) \cdot \bar{g} \mid \delta \in [0, 1]$$

## Greyness

GST includes greyness as an uncertainty measurement. With higher values of greyness then a higher uncertainty degree.

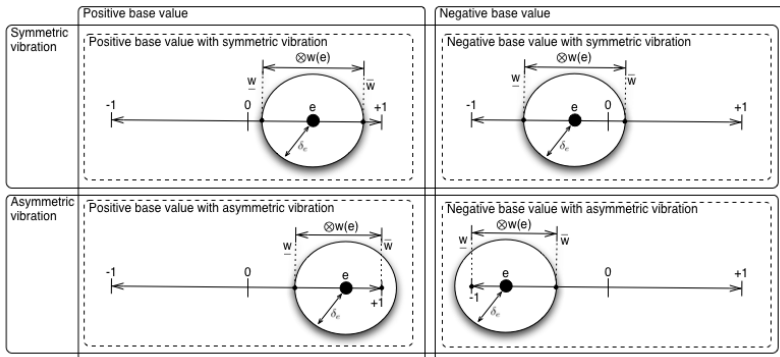
$$\phi(\otimes g) = \frac{|\ell(\otimes g)|}{\ell(\otimes \psi)}$$



# Grey weight with base value and symmetric / asymmetric vibration

$$\otimes w(e) \in [\underline{w}, \bar{w}]$$

$$\otimes w(e) \in \begin{cases} [e - \delta_e, e + \delta_e] & \text{if } (-1 \leq e - \delta_e \leq +1) \wedge (-1 \leq e + \delta_e \leq +1) \\ [e - \delta_e, +1] & \text{if } (-1 \leq e - \delta_e \leq +1) \wedge (+1 < e + \delta_e) \\ [-1, e + \delta_e] & \text{if } (e - \delta_e < -1) \wedge (-1 \leq e + \delta_e \leq +1) \\ [-1, +1] & \text{if } (e - \delta_e < -1) \wedge (+1 < e + \delta_e) \end{cases}$$



# Grey weight with same base value and different (+|−) vibration

## Vibration illustrative example

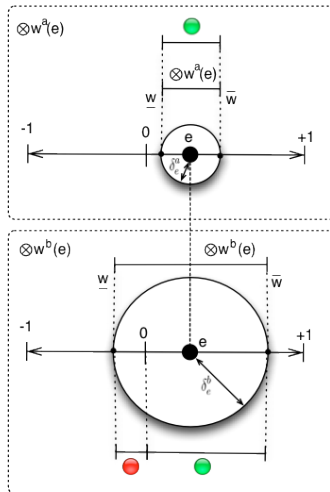
- Same base value ( $e$ ) with a couple of different vibrations ( $\delta_e^a$  and  $\delta_e^b$ )

### $\otimes w^a(e)$

- Base value positive and  $e \pm \delta_e^a > 0$

### $\otimes w^b(e)$

- Base value positive and  $\underline{w}^b < 0 \wedge \overline{w}^b > 0$
- Note that base value is the same in both cases ( $\otimes w^a(e)$  and  $\otimes w^b(e)$ )
- The vibration includes a negative zone within the grey number  $[e - \delta_e^b, 0]$



# FGCM uncertainty measures

## Greyness

Furthermore, FGCM includes greyness as an uncertainty measurement. Higher values of greyness mean that the results have a higher uncertainty degree. It is computed as follows

$$\phi(\otimes c_i) = \frac{|\ell(\otimes c_i)|}{\ell(\otimes \psi)}$$

where  $|\ell(\otimes c_i)|$  is the absolute value of the length of grey node  $\otimes c_i$  state value, and  $\ell(\otimes \psi)$  is the absolute value of the range in the information space, denoted by  $\psi$ .

FGCM maps the nodes' states within an interval  $[0, 1]$  or  $[-1, +1]$  if negative values are allowed. In this sense,

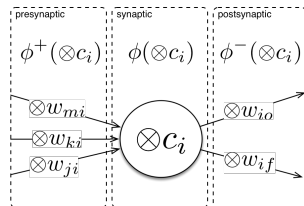
$$\ell(\otimes \psi) = \begin{cases} 1 & \text{if } \{\otimes c_i, \otimes w_i\} \subseteq [0, 1] \\ 2 & \text{if } \{\otimes c_i, \otimes w_i\} \subseteq [-1, +1] \end{cases} \quad \begin{matrix} \forall \otimes c_i, \otimes w_i \\ \forall \otimes c_i, \otimes w_i \end{matrix}$$

# Static

## Grey measures

- FGCM includes new uncertainty measures

Greyiness in-degree	$\phi^+(\otimes c_j) = \sum_{i=1}^n \frac{ \ell(\otimes w_{ij}) }{\ell(\otimes \psi)}$
Greyiness out-degree	$\phi^-(\otimes c_j) = \sum_{i=1}^n \frac{ \ell(\otimes w_{ij}) }{\ell(\otimes \psi)}$
Overall greyiness	$\phi^\oplus(\otimes c_i) = \phi(\otimes c_i) + \phi^+(\otimes c_j) + \phi^-(\otimes c_j)$



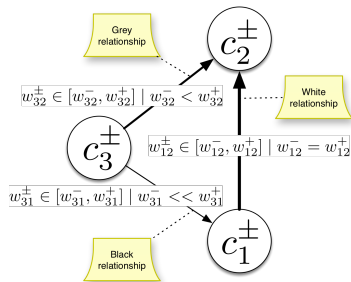
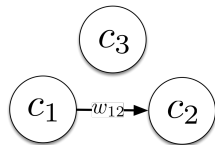
# FCM vs. FGCM

## Fuzzy Cognitive Maps

- 1 Just one kind of relationship between nodes  
 $w_{13}$

## Fuzzy Grey Cognitive Maps

- 1 White relationship  
 $\otimes w_{13} \in [\underline{w}_{13}, \overline{w}_{13}] \mid \underline{w}_{13} = \overline{w}_{13}$
- 2 Grey relationship  
 $\otimes w_{12} \in [\underline{w}_{12}, \overline{w}_{12}] \mid \underline{w}_{12} < \overline{w}_{12}$
- 3 Black relationship  
 $\otimes w_{23} \in [\underline{w}_{23}, \overline{w}_{23}] \mid \underline{w}_{23} = -1 \wedge \overline{w}_{23} = +1$



# FGCM dynamics

## Initial grey vector state

FGCMs dynamics begins with the design of the initial grey vector state  $\otimes \tilde{c}(0)$ , which represents a proposed initial grey stimuli.

## Grey activation function

The updated nodes' states are computed in an iterative inference process with a grey activation function.

## Grey unipolar sigmoid

The unipolar sigmoid function is the most used one in FCM and FGCM when the concept value maps in the range  $[0, 1]$ .

## Grey hyperbolic tangent

When the concepts' states map in the range  $[-1, +1]$  the function used would be the hyperbolic tangent.

$$\begin{aligned}\otimes \tilde{c}(0) &= (\otimes c_1(0) \quad \dots \quad \otimes c_n(0)) \\ &= ([\underline{c}_1(0), \bar{c}_1(0)] \quad \dots \quad [\underline{c}_n(0), \bar{c}_n(0)])\end{aligned}$$

$$\begin{aligned}\otimes c_j(t+1) &\in f\left(\otimes c_j(t) + \sum_{i=1}^N w_{ij} \cdot \otimes c_j(t)\right) \\ &\in f(\otimes c_j(t*)) \\ &\in \left[f(\underline{c}_j(t*)), f(\bar{c}_j(t*))\right] \\ &\in [\underline{c}_j(t+1), \bar{c}_j(t+1)]\end{aligned}$$

$$\otimes c_i(t+1) \in \left[\left(1 + e^{-\lambda \cdot \underline{c}_i(t*)}\right)^{-1}, \left(1 + e^{-\lambda \cdot \bar{c}_i(t*)}\right)^{-1}\right]$$

$$\otimes c_i(t+1) \in \left[\left(\frac{e^{2 \cdot \lambda \cdot \underline{c}_i(t*)} - 1}{e^{2 \cdot \lambda \cdot \underline{c}_i(t*)} + 1}\right), \left(\frac{e^{2 \cdot \lambda \cdot \bar{c}_i(t*)} - 1}{e^{2 \cdot \lambda \cdot \bar{c}_i(t*)} + 1}\right)\right]$$

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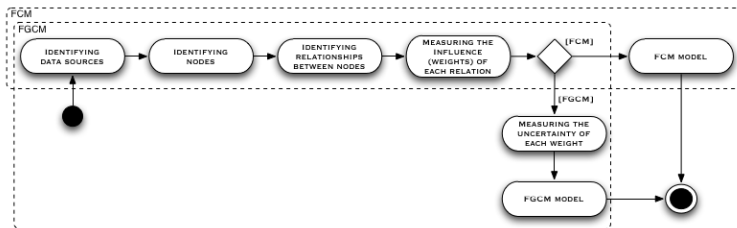
## FGCM stability

- The FGCM inference process finish when the stability is reached.
- After its inference process, the FGCM reaches either one stable state following a number of iterations.
  - 1 It settles down to a fixed pattern of node states, the so-called **grey hidden pattern or grey fixed-point attractor**.
  - 2 Moreover, the state could keep cycling between several fixed states, known as a **limit grey cycle**.
  - 3 Using a continuous activation function, a third state would be a **grey chaotic attractor**. It happens when, instead of stabilizing, the FGCM continues to produce different grey vector states for each iteration.

## FCM vs. FGCM Construction process

### Grey Weights determination

- 1 Base value definition. It could be assign as the same way than in FCM. Anyway, the base value is located within the FGCM nodes/weights' range  $w_{ij}^e \in \{[0, +1] \mid [-1, +1]\}$ .
- 2 Selection of the vibration value of the base value  $\delta_e$ . If construction is automatic, vibration could represent noise or another measurements distortions. In the experts based construction process the base value determine the  $\delta_e$  value.
  - If the expert has the whole trust on the base value  $e$ , the vibration's value would be  $\delta_e = 0$ .
  - if the expert has not any trust on the base value, the vibration's value would be as  $w_{ij}^e \pm \delta_e = \pm 1$ .
  - Otherwise,  $(w_{ij}^e + \delta_e \leq +1) \wedge (w_{ij}^e - \delta_e \geq -1)$ .



# Applications

Knowledge-Based Systems 30 (2012) 151–160



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Contents lists available at SciVerse ScienceDirect

Knowledge-Based Systems

journal homepage: [www.elsevier.com/locate/knosys](http://www.elsevier.com/locate/knosys)



## A Fuzzy Grey Cognitive Maps-based Decision Support System for radiotherapy treatment planning

Jose L. Salmeron<sup>a,\*</sup>, Elpiniki I. Papageorgiou<sup>b</sup>

<sup>a</sup> University Pablo de Olavide, School of Engineering, 1st km. Utrera Road, 41013 Seville, Spain

<sup>b</sup> Technological Educational Institute of Lamia, Department of Informatics and Computer Technology, 3rd km. Old National Road Lamia-Athens, TK 35100 Lamia, Greece

Applied Soft Computing 12 (2012) 3818–3824



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Contents lists available at SciVerse ScienceDirect

Applied Soft Computing

journal homepage: [www.elsevier.com/locate/asoc](http://www.elsevier.com/locate/asoc)



## Fuzzy Grey Cognitive Maps in reliability engineering

Jose L. Salmeron<sup>a</sup>, Ester Gutierrez<sup>b,\*</sup>

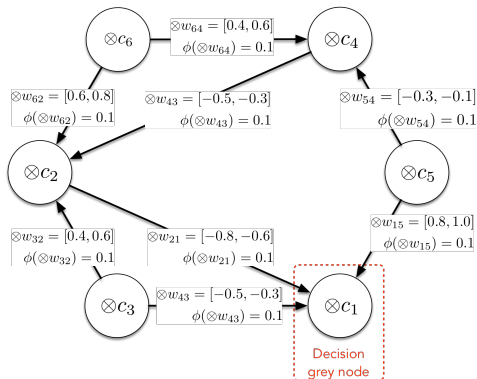
<sup>a</sup> University Pablo de Olavide, 1st km. Utrera Road, 41013 Seville, Spain

<sup>b</sup> University of Seville, C. Descubrimientos s/n, 41092 Seville, Spain

# Model

## Problem

- A synthetic case study for showing the added value of FGCMs.
- The simulation consists in an Intelligent Security System controlling several surveillance assets.



# Model

## Problem

- A synthetic case study for showing the added value of FGCMs.
- The simulation consists in an Intelligent Security System controlling several surveillance assets.

## Solution

$$\blacksquare \otimes c_1(s)_2 \succ \otimes c_1(s)_3 \succ \otimes c_1(s)_1$$

Table: Initial grey vector states  $\{\otimes c(0)_i\}_{i=1}^3$

	$\otimes c(0)_1$	$\otimes c(0)_2$	$\otimes c(0)_3$
$\otimes c_1(0)$	[.0, .0]	[.0, .0]	[.0, .0]
$\otimes c_2(0)$	[.0, .0]	[.0, .0]	[.0, .0]
$\otimes c_3(0)$	[.4, .4]	[.0, .0]	[.0, .0]
$\otimes c_4(0)$	[.0, .0]	[.0, .0]	[.0, .0]
$\otimes c_5(0)$	[-.5, -.5]	[-.5, -.5]	[-.8, -.8]
$\otimes c_6(0)$	[.6, .6]	[-.4, -.4]	[.0, .0]

$$\otimes c_1(s)_1 \in [-.9420, -.8909]$$

$$\otimes c_1(s)_2 \in [.8240, .9043]$$

$$\otimes c_1(s)_3 \in [.7720, .8848]$$

# Outline

## 1 FCM fundamentals

- Basics
- Construction
- Analysis

## 2 Fuzzy Grey Cognitive Maps

- Grey Systems Theory
- FGCM fundamentals
- Analysis
- Example: A FGCM-based intelligent system

## 3 Machine Learning in Grey-based Soft Computing techniques

- Unsupervised Learning
- Supervised Learning

# Learning

International Journal of Approximate Reasoning 53 (2012) 54–65



Contents lists available at SciVerse ScienceDirect

## International Journal of Approximate Reasoning

journal homepage: [www.elsevier.com/locate/ijar](http://www.elsevier.com/locate/ijar)



## Learning Fuzzy Grey Cognitive Maps using Nonlinear Hebbian-based approach

Elpiniki I. Papageorgiou<sup>a,\*</sup>, Jose L. Salmeron<sup>b</sup>

<sup>a</sup> *Technological Educational Institute of Lamia, Department of Informatics and Computer Technology, 3rd km. Old National Road Lamia-Athens, TK 35100 Lamia, Greece*

<sup>b</sup> *University Pablo de Olavide, School of Engineering, 1st km. Utrera Road, 41013 Seville, Spain*

## NHL



E.I. Papageorgiou and J.L. Salmeron (2011). Learning Fuzzy Grey Cognitive Maps using Nonlinear Hebbian-based approach. International Journal of Approximate Reasoning, 53(1), pp. 54-65.

## NHL-FGCM

- NHL learning rule for FGCMs introduces a learning rate parameter, the determination of input and output nodes, and the termination conditions.

$$\Delta \otimes w_{ji}(k) = \eta_k \cdot \otimes c_j(k-1) \cdot (\otimes c_i(k-1) - \otimes c_j(k-1) \cdot \otimes w_{ji}(k-1))$$

where the coefficient  $\eta_k$  is a very small positive scalar factor called learning parameter.

- This simple rule states that if  $c_i(k)$  is the value of node  $c_i$  at iteration  $k$ , and  $c_j$  is the value of the triggering node  $c_j$  which triggers the node  $c_i$ , the corresponding grey weight  $\otimes w_{ji}$  from node  $c_j$  towards the node  $c_i$  is increased proportional to their product multiplied with the learning rate parameter minus the grey weight decay at iteration step  $k$ .

As a result

$$\otimes w_{ji}(k) = \left[ \overbrace{\otimes w_{ji}(k-1) + \eta_k \cdot \otimes c_j \cdot (\otimes c_i(k) - \otimes c_j \cdot \otimes w_{ji}(k-1))}^{\underline{w}_{ji}(k)}, \right. \\ \left. \underbrace{\overline{w}_{ij}(k-1) + \eta_k \cdot \otimes c_j \cdot (\otimes c_i(k) - \otimes c_j \cdot \otimes w_{ji}(k-1))}_{\overline{w}_{ji}(k)} \right]$$

# NHL



E.I. Papageorgiou and J.L. Salmeron (2011). Learning Fuzzy Grey Cognitive Maps using Nonlinear Hebbian-based approach. *International Journal of Approximate Reasoning*, 53(1), pp. 54-65.

## NHL-FGCM termination criteria

These criteria determine when the iterative process of the learning algorithm terminates. Through this process and when the termination conditions are met, the final weight matrix updated  $\otimes w^{updated}$  is obtained.

- 1 Maximization of the objective function  $J$ , which has been defined by Hebb's rule.

$$\begin{aligned} &\text{maximize } J = E\{z^2\} \\ &\text{subject to: } ||\mathbf{w}|| = 1 \end{aligned}$$

where  $z = f(y)$ , and  $f$  is the sigmoid function. The objective function  $J$  is defined as  $J = \sum_{i=1}^l (OC_i)^2$ , where  $l$  is the number of  $OC$ s.

- 2 Minimization of the variation of two subsequent values of  $OC$ s.

$$|OC_j^{k+1} - OC_j^k| < e$$

where the term  $e$  is a tolerance level keeping the variation of values of  $OC(s)$  as low as possible and it is proposed as  $e = 0.001$ .

- 3 Stability of the grey vector state.

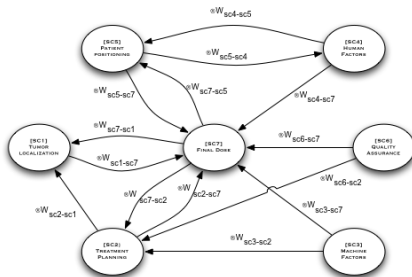
# NHL



E.I. Papageorgiou and J.L. Salmeron (2011). Learning Fuzzy Grey Cognitive Maps using Nonlinear Hebbian-based approach. International Journal of Approximate Reasoning. forthcoming.

## NHL-FGCM Supervisor

Node	Description
SC1	Tumor Localization.
SC2	Dose prescribed from Treatment Planning (TPD).
SC3	Machine factors.
SC4	Human factors.
SC5	Patient positioning and immobilization.
SC6	Quality Assurance (QA).
SC7	Final Dose given to the target volume (FD).



# RCGA-FGCM

International Journal of Approximate Reasoning 55 (2014) 1319–1335



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Contents lists available at [ScienceDirect](#)

## International Journal of Approximate Reasoning

[www.elsevier.com/locate/ijar](http://www.elsevier.com/locate/ijar)



### Evolutionary learning of fuzzy grey cognitive maps for the forecasting of multivariate, interval-valued time series ☆



Wojciech Froelich<sup>a,\*</sup>, Jose L. Salmeron<sup>b,\*\*</sup>

<sup>a</sup> Institute of Computer Science, University of Silesia, ul. Bedzinska 39, Sosnowiec, Poland

<sup>b</sup> Computational Intelligence Lab, University Pablo de Olavide, 1st km. Utrera road, 41013 Seville, Spain

# RCGA-FGCM



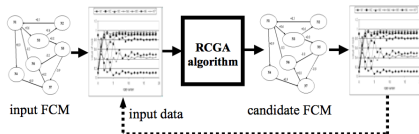
W. Froelich and J.L. Salmeron (2014). Evolutionary Learning of Fuzzy Grey Cognitive Maps for the Forecasting of Multivariate, Interval-Valued Time Series. International Journal of Approximate Reasoning 55(6), pp. 1319-1335.

## Population

- Data sources (final states' set vs. time series)
- The objective is to optimize the matrix  $[\otimes A_{n \times n}]$  with respect to the forecasting accuracy.

## Fitness

- Fitness function depends on raw data



## Algorithm 1: RCGA-based FGCM pseudo-code

**Data:** Domain's raw data

**Result:** Automatic built FCM

```

1 Choose initial FGCM population (random);
2 Design fitness function;
3 Evaluate each FGCM's fitness;
4 while Termination is false do
5     Select best-ranking FGCM to reproduce;
6     Mate pairs at random;
7     Prune FGCM population;
8     Crossover operator;
9     Mutation operator;
10    Evaluate each FGCM's fitness;
11    Check termination criteria;
12 end
13 Select best-fitness FGCM
    
```

## More ...

### More ideas

- FGCM learning and automatic construction with new algorithms
- FGCM in control systems
- FGCM in biomedical engineering
- FGCM in environmental control
- FGCM synaptic plasticity
- and so on
- . . .



The Leverhulme Trust

# International Network on Grey Systems and its Applications

**Keynote talk:  
Machine Learning in Grey-Based Soft Computing  
techniques**

Keynote speaker: **Prof. Jose L. Salmeron**

Universidad Pablo de Olavide (Seville, Spain)

September 23, 2015